Shipboard Three-Phase Power Transformer Analysis

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The cover photo depicts a delta-delta transformer bank onboard ex USS Midway (CV-41). Each single-phase transformer in the bank is rated for 94 kVA. The primary is rated at 450 volts and the secondary at 120 volts, 60 Hz. ac power.

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1. Introduction

This document analyzes three phase transformer banks that are applicable to shipboard power systems: delta-wye, delta-delta, wye-wye, wye-delta, and multiphase. Conclusions are made based on the results of the analysis. The analysis is conducted symbolically using wxMaxima version 22.04.0. then simplified manually. All equations should be verified before use in the case an inadvertent error was introduced during simplification.

All equations are preceded with the chapter number and a dash. In each chapter, the system equations are numbered beginning with 101. Changes in system topology are implemented by changes to the appropriate numbered system equation and provided with a letter suffix. The system variables (currents, voltages, magnetomotive force, and magnetic flux) are expressed in terms of parameters and are numbered beginning with 1. Simplifications and changes in topology are indicated by a letter suffix. Other equations are numbered beginning with 201.

The information contained herein is provided in the hope that it will be useful, but WITHOUT ANY WARRANTY; without even the implied warranty of MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. Readers should follow all applicable laws, regulations and class society rules in the design and operation of shipboard power system apparatus.

Please contact the author if you find any errors. Also, feel free to suggest improvements or additions to this document. Revisions are intended to be produced as needed to correct errors and provide additional material. For the latest revision check http://doerry.org/papers/papers.htm.

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2. Single-Phase Transformers

Three-phase transformer banks are often created using three single phase transformers. A single-phase transformer is depicted in Figure 1 where the subscript "*x*" refers to one of the phases of the three-phase transformer (*a, b,* or *c*).

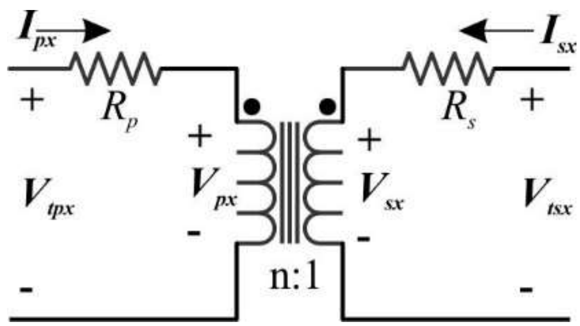


Figure 1: Single Phase Transformer

The constitutive equations for a single-phase transformer are given by:

$$V_{px} = nV_{sx} \quad [2\text{-}101]$$

$$nI_{px} = -I_{sx} \quad [2\text{-}102]$$

$$V_{tpx} = V_{px} + R_p I_{px} \quad [2\text{-}103]$$

$$V_{tsx} = V_{sx} + R_s I_{sx} \quad [2\text{-}104]$$

Through substitution, two variables can be eliminated:

$$V_{tpx} = n\left(V_{tsx} + R_s n I_{px}\right) + R_p I_{px} \quad [2\text{-}105]$$

Restating

$$V_{tpx} = nV_{tsx} + \left(R_p + R_s\, n^2\right) I_{px} \quad [2\text{-}106]$$

Let

$$R_T = R_p + R_s\, n^2 \quad [2\text{-}107]$$

Then:

$$V_{tpx} = nV_{tsx} + R_T I_{px} \quad [2\text{-}108]$$

$$I_{px} = -\frac{1}{n} I_{sx} \quad [2\text{-}109]$$

An alternate form based on the secondary variables is given by:

$$V_{tsx} = \frac{1}{n} V_{tpx} + \frac{1}{n^2} R_T I_{sx} \quad [2\text{-}110]$$

$$I_{sx} = -nI_{px} \quad [2\text{-}111]$$

These equations are used in the subsequent chapters for the various three-phase transformer variants.

Note that while R_p, R_s, and R_t as real numbers represent resistances, if they are complex numbers, they can represent a generalized impedance. In all equations, one can substitute R_p, R_s, and R_t with Z_p, Z_s, and Z_t.

3. Delta-Wye Transformers

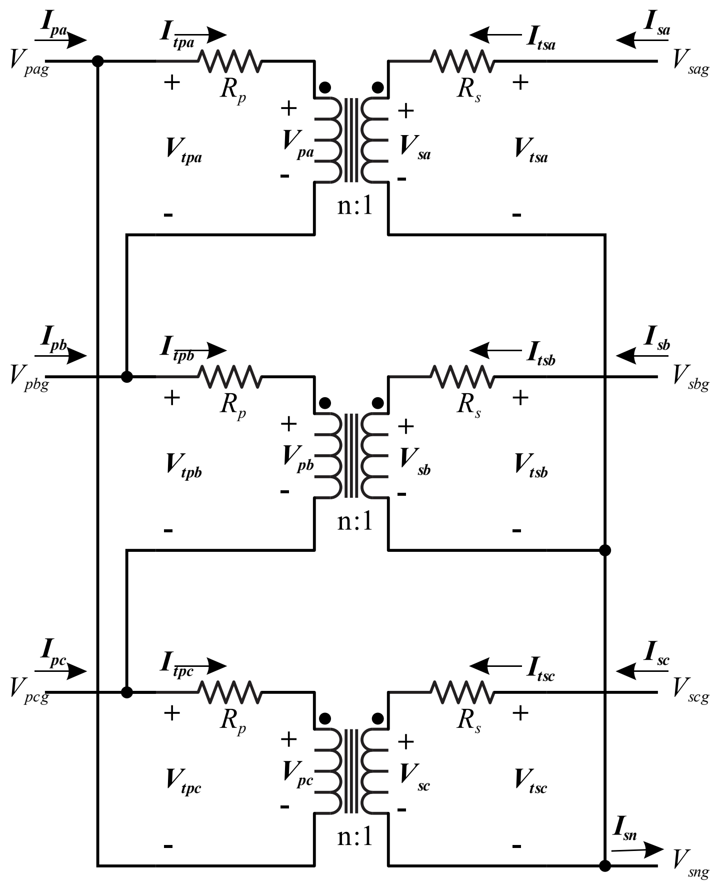
Three phase delta-wye transformers are typically used in shipboard power systems where the secondary is part of a high resistance grounded or solidly grounded distribution system. This document explores the steady-state performance of three "nearly ideal" transformers (as described in Chapter 2) when connected into a delta-wye transformer bank. Each transformer is assumed to be an ideal transformer with each transformer winding having a series resistance. If this resistance is assumed to have a complex value, it can represent a general impedance. The delta-wye transformer configurations are provided with ideal voltage sources and with resistive loads. The resistive load value may also be complex. A resistance to ground from the wye neutral conductor of the secondary is modeled, but its value can be set to zero in cases where the secondary system is solidly grounded. Cases with unbalanced load resistances and with ground faults are analyzed.

Figure 2 depicts the schematic for a delta-wye connected transformer as well as the typical schematic symbol used in many power system drawings. In the schematic symbol, thick parallel lines represent the primary and secondary windings of an equivalent (or actual) single phase transformer. As described in Chapter 2, the resistances in the primary (R_p) and secondary (R_s) can be combined into a total resistance (R_T) where n is the turns ratio of the single-phase transformers.

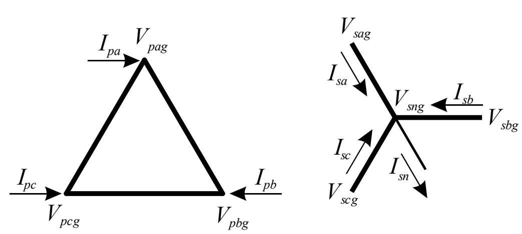
$$R_T = R_p + R_s\, n^2 \qquad [2\text{-}107]$$

The turns ratio n is for each of the individual transformers. The relationship of the line-to-line voltages (for balanced three phase voltages) of the primary (V_{pll}) and secondary (V_{sll}) circuits is given by:

$$V_{pll} = V_{pag} - V_{pbg} = n\left(V_{sag} - V_{sng}\right) = n\left(\frac{V_{sll}}{\sqrt{3}}\right) = \left(\frac{n}{\sqrt{3}}\right)V_{sll} \qquad [3\text{-}201]$$



(a)



(b)

Figure 2: Delta-wye transformer – (a) circuit diagram (b) schematic.

The voltages and currents of the primary and secondary windings are related.

$$I_{pa} + I_{tpc} - I_{tpa} = 0 \quad [3\text{-}101]$$

$$I_{pb} + I_{tpa} - I_{tpb} = 0 \quad [3\text{-}102]$$

$$I_{pc} + I_{tpb} - I_{tpc} = 0 \quad [3\text{-}103]$$

$$nI_{tpa} + I_{tsa} = 0 \quad [3\text{-}104]$$

$$nI_{tpb} + I_{tsb} = 0 \quad [3\text{-}105]$$

$$nI_{tpc} + I_{tsc} = 0 \quad [3\text{-}106]$$

$$R_T I_{tpa} - V_{tpa} + nV_{tsa} = 0 \quad [3\text{-}107]$$

$$R_T I_{tpb} - V_{tpb} + nV_{tsb} = 0 \quad [3\text{-}108]$$

$$R_T I_{tpc} - V_{tpc} + nV_{tsc} = 0 \quad [3\text{-}109]$$

$$V_{tpa} - V_{pag} + V_{pbg} = 0 \quad [3\text{-}110]$$

$$V_{tpb} - V_{pbg} + V_{pcg} = 0 \quad [3\text{-}111]$$

$$V_{tpc} - V_{pcg} + V_{pag} = 0 \quad [3\text{-}112]$$

$$I_{sa} - I_{tsa} = 0 \quad [3\text{-}113]$$

$$I_{sb} - I_{tsb} = 0 \quad [3\text{-}114]$$

$$I_{sc} - I_{tsc} = 0 \quad [3\text{-}115]$$

$$V_{tsa} - V_{sag} + V_{sng} = 0 \quad [3\text{-}116]$$

$$V_{tsb} - V_{sbg} + V_{sng} = 0 \quad [3\text{-}117]$$

$$V_{tsc} - V_{scg} + V_{sng} = 0 \quad [3\text{-}118]$$

While the equations above describe the relationships among the transformer terminals, the following equations are included to fully define the circuit shown in Figure 3. This figure includes a three-phase source with the neutral grounded with a resistor. It also includes a 4-wire load on the secondary system in addition to a grounding resistor connected to the secondary neutral conductor. Each of the load resistances may be different.

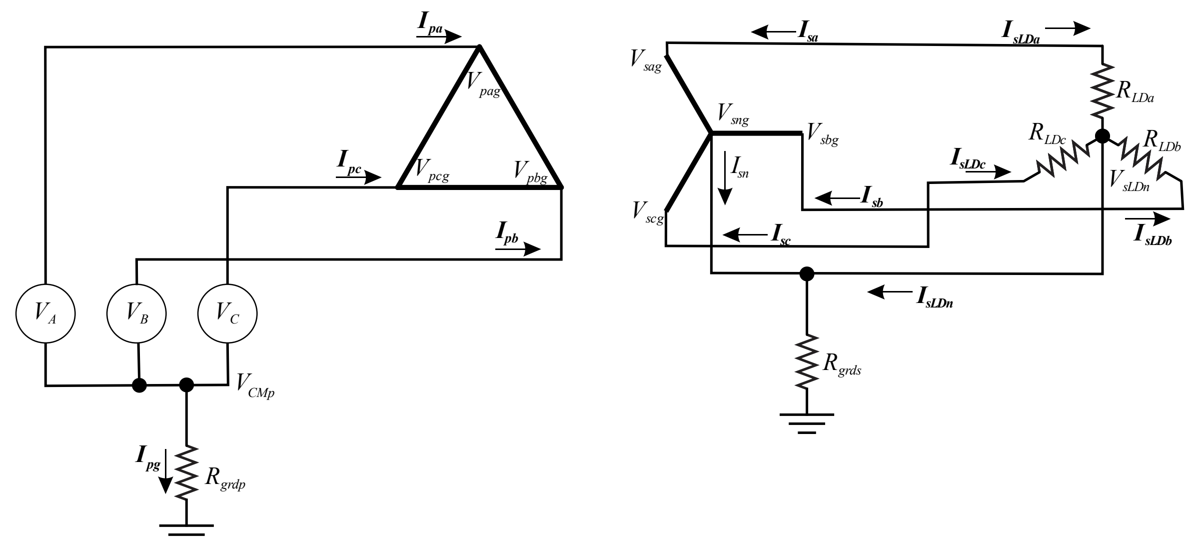


Figure 3: Delta-wye transformer with source and load

Assume a three-phase source (V_A, V_B and V_C are assumed to be sinusoidal and of the same frequency, but the equations and solution below only requires them to be ac)

$$V_{pag} - V_{CMp} = V_A \quad [3\text{-}119]$$

$$V_{pbg} - V_{CMp} = V_B \quad [3\text{-}120]$$

$$V_{pcg} - V_{CMp} = V_C \quad [3\text{-}121]$$

Assume a grounding resistor at the source

$$R_{grdp} I_{pg} - V_{CMp} = 0 \quad [3\text{-}122]$$

$$I_{pg} + I_{pa} + I_{pb} + I_{pc} = 0 \quad [3\text{-}123]$$

Assume a wye connected load where each of the load resistances may be different

$$V_{sag} - V_{sLDn} - I_{sLDa} R_{LDa} = 0 \quad [3\text{-}124]$$

$$V_{sbg} - V_{sLDn} - I_{sLDb} R_{LDb} = 0 \quad [3\text{-}125]$$

$$V_{scg} - V_{sLDn} - I_{sLDc} R_{LDc} = 0 \quad [3\text{-}126]$$

$$-I_{sLDn} + I_{sLDa} + I_{sLDb} + I_{sLDc} = 0 \quad [3\text{-}127]$$

Account for the grounding resistor

$$V_{sng} - (I_{sn} + I_{sLDn}) R_{grds} = 0 \quad [3\text{-}128]$$

$$-I_{sn} + I_{sa} + I_{sb} + I_{sc} = 0 \quad [3\text{-}129]$$

Relate the secondary currents

$$I_{sa} + I_{sLDa} = 0 \quad [3\text{-}130]$$

$$I_{sb} + I_{sLDb} = 0 \quad [3\text{-}131]$$

$$I_{sc} + I_{sLDca} = 0 \quad [3\text{-}132]$$

Equate the neutral conductor voltages of secondary and load

$$V_{sng} - V_{sLDn} = 0 \quad [3\text{-}133]$$

Solving this set of equations results in

$$I_{pa} = \frac{(-R_{LDa}V_C - R_{LDc}V_B + (R_{LDc} + R_{LDa})V_A)n^2 + R_T(-V_C - V_B + 2V_A)}{R_{LDa}R_{LDc}n^4 + (R_{LDc} + R_{LDa})R_T n^2 + R_T{}^2} \quad [3\text{-}1]$$

$$I_{pb} = -\frac{(R_{LDa}V_C + (-R_{LDb} - R_{LDa})V_B + R_{LDb}V_A)n^2 + R_T(V_C - 2V_B + V_A)}{R_{LDa}R_{LDb}n^4 + (R_{LDb} + R_{LDa})R_T n^2 + R_T{}^2} \quad [3\text{-}2]$$

$$I_{pc} = -\frac{((-R_{LDc} - R_{LDb})V_C + R_{LDc}V_B + R_{LDb}V_A)n^2 + R_T(-2V_C + V_B + V_A)}{R_{LDb}R_{LDc}n^4 + (R_{LDc} + R_{LDb})R_T n^2 + R_T{}^2} \quad [3\text{-}3]$$

$$I_{tpa} = \frac{V_A - V_B}{R_{LDa}n^2 + R_T} \quad [3\text{-}4]$$

$$I_{tpb} = \frac{V_B - V_C}{R_{LDb}n^2 + R_T} \quad [3\text{-}5]$$

$$I_{tpc} = -\frac{V_A - V_C}{R_{LDc}n^2 + R_T} \quad [3\text{-}6]$$

$$V_{pag} = V_A \quad [3\text{-}7]$$

$$V_{pbg} = V_B \tag{3-8}$$

$$V_{pcg} = V_C \tag{3-9}$$

$$V_{tpa} = V_A - V_B \tag{3-10}$$

$$V_{tpb} = V_B - V_C \tag{3-11}$$

$$V_{tpc} = V_C - V_A \tag{3-12}$$

$$I_{sa} = -\frac{(V_A - V_B)n}{R_{LDa}n^2 + R_T} \tag{3-13}$$

$$I_{sb} = -\frac{(V_B - V_C)n}{R_{LDb}n^2 + R_T} \tag{3-14}$$

$$I_{sc} = \frac{(V_A - V_C)n}{R_{LDc}n^2 + R_T} \tag{3-15}$$

$$I_{tsa} = -\frac{(V_A - V_B)n}{R_{LDa}n^2 + R_T} \tag{3-16}$$

$$I_{tsb} = -\frac{(V_B - V_C)n}{R_{LDb}n^2 + R_T} \tag{3-17}$$

$$I_{tsc} = \frac{(V_A - V_C)n}{R_{LDc}n^2 + R_T} \tag{3-18}$$

$$V_{sag} = \frac{(R_{LDa}V_A - R_{LDa}V_B)n}{R_{LDa}n^2 + R_T} \tag{3-19}$$

$$V_{sbg} = \frac{(R_{LDb}V_B - R_{LDb}V_C)n}{R_{LDb}n^2 + R_T} \tag{3-20}$$

$$V_{scg} = -\frac{(R_{LDc}V_A - R_{LDc}V_C)n}{R_{LDc}n^2 + R_T} \tag{3-21}$$

$$V_{tsa} = \frac{(R_{LDa}V_A - R_{LDa}V_B)n}{R_{LDa}n^2 + R_T} \tag{3-22}$$

$$V_{tsb} = \frac{(R_{LDb}V_B - R_{LDb}V_C)n}{R_{LDb}n^2 + R_T} \tag{3-23}$$

$$V_{tsc} = -\frac{(R_{LDc}V_A - R_{LDc}V_C)n}{R_{LDc}n^2 + R_T} \tag{3-24}$$

$$V_{CMp} = 0 \tag{3-25}$$

$$I_{pg} = 0 \tag{3-26}$$

$$V_{sng} = 0 \tag{3-27}$$

$$I_{sn} = \frac{(R_{LDa}(R_{LDc} - R_{LDb})V_C + (R_{LDb}R_{LDc} - R_{LDa}R_{LDc})V_B + (R_{LDa}R_{LDb} - R_{LDb}R_{LDc})V_A)n^5}{R_{LDa}R_{LDb}R_{LDc}n^6 + (R_{LDa}(R_{LDc} + R_{LDb}) + R_{LDb}RLDc)R_Tn^4 + (R_{LDc} + R_{LDb} + R_{LDc})R_T{}^2n^2 + R_T{}^3} + \frac{R_T\big((R_{LDc} - R_{LDb})V_C + (R_{LDb} - R_{LDa})V_B + (R_{LDa} - R_{LDc})V_A\big)n^3}{R_{LDa}R_{LDb}R_{LDc}n^6 + (R_{LDa}(R_{LDc} + R_{LDb}) + R_{LDb}RLDc)R_Tn^4 + (R_{LDc} + R_{LDb} + R_{LDc})R_T{}^2n^2 + R_T{}^3} \tag{3-28}$$

$$I_{sLDa} = \frac{(V_A - V_B)n}{R_{LDa}n^2 + R_T} \tag{3-29}$$

$$I_{sLDb} = \frac{(V_B - V_C)n}{R_{LDb}n^2 + R_T} \quad [3\text{-}30]$$

$$I_{sLDc} = -\frac{(V_A - V_C)n}{R_{LDc}n^2 + R_T} \quad [3\text{-}31]$$

$$I_{sLDn} = -I_{sn} \quad [3\text{-}32]$$

$$V_{sLDn} = 0 \quad [3\text{-}33]$$

If we make the assumption that R_T is negligible, then several of the solutions may be simplified:

$$I_{pa} = \frac{(-R_{LDa}V_C - R_{LDc}V_B + (R_{LDc} + R_{LDa})V_A)}{R_{LDa}R_{LDc}n^2} \quad [3\text{-}1a]$$

$$I_{pb} = -\frac{(R_{LDa}V_C + (-R_{LDb} - R_{LDa})V_B + R_{LDb}V_A)}{R_{LDa}R_{LDb}n^2} \quad [3\text{-}2a]$$

$$I_{pc} = -\frac{((-R_{LDc} - R_{LDb})V_C + R_{LDc}V_B + R_{LDb}V_A)}{R_{LDb}R_{LDc}n^2} \quad [3\text{-}3a]$$

$$I_{tpa} = \frac{V_A - V_B}{R_{LDa}n^2} \quad [3\text{-}4a]$$

$$I_{tpb} = \frac{V_B - V_C}{R_{LDb}n^2} \quad [3\text{-}5a]$$

$$I_{tpc} = -\frac{V_A - V_C}{R_{LDc}n^2} \quad [3\text{-}6a]$$

$$I_{sa} = -\frac{(V_A - V_B)}{R_{LDa}n} \quad [3\text{-}13a]$$

$$I_{sb} = -\frac{(V_B - V_C)}{R_{LDb}n} \quad [3\text{-}14a]$$

$$I_{sc} = \frac{(V_A - V_C)}{R_{LDc}n} \quad [3\text{-}15a]$$

$$I_{tsa} = -\frac{(V_A - V_B)}{R_{LDa}n} \quad [3\text{-}16a]$$

$$I_{tsb} = -\frac{(V_B - V_C)}{R_{LDb}n} \quad [3\text{-}17a]$$

$$I_{tsc} = \frac{(V_A - V_C)}{R_{LDc}n} \quad [3\text{-}18a]$$

$$V_{sag} = \frac{(V_A - V_B)}{n} \quad [3\text{-}19a]$$

$$V_{sbg} = \frac{(V_B - V_C)}{n} \quad [3\text{-}20a]$$

$$V_{scg} = \frac{(V_C - V_A)}{n} \quad [3\text{-}21a]$$

$$V_{tsa} = \frac{(V_A - V_B)}{n} \quad [3\text{-}22a]$$

$$V_{tsb} = \frac{(V_B - V_C)}{n} \quad [3\text{-}23a]$$

$$V_{tsc} = \frac{(V_C - V_A)}{n} \quad [3\text{-}24a]$$

$$I_{sn} = \frac{((R_{LDa}R_{LDc} - R_{LDa}R_{LDb})V_C + (R_{LDb}R_{LDc} - R_{LDa}R_{LDc})V_B + (R_{LDa}R_{LDb} - R_{LDb}R_{LDc})V_A)}{R_{LDa}R_{LDb}R_{LDc}n} \quad [3\text{-}28a]$$

$$I_{sLDa} = \frac{(V_A - V_B)}{R_{LDa}n} \quad [3\text{-}29a]$$

$$I_{sLDb} = \frac{(V_B - V_C)}{R_{LDb}n} \quad [3\text{-}30a]$$

$$I_{sLDc} = \frac{(V_C - V_A)}{R_{LDc}n} \quad [3\text{-}31a]$$

The average of the transformer primary winding currents is given by:

$$\frac{I_{tpa} + I_{tpb} + I_{tpc}}{3} = \frac{1}{3}\left(\frac{V_A - V_B}{R_{LDa}n^2} + \frac{V_B - V_C}{R_{LDb}n^2} + \frac{V_C - V_A}{R_{LDc}n^2}\right) \quad [3\text{-}202]$$

$$\frac{I_{tpa} + I_{tpb} + I_{tpc}}{3} = \frac{R_{LDc}R_{LDb}(V_A - V_B) + R_{LDa}R_{LDc}(V_B - V_C) + R_{LDa}R_{LDb}(V_C - V_A)}{3R_{LDa}R_{LDb}R_{LDc}n^2} \quad [3\text{-}203]$$

$$\frac{I_{tpa} + I_{tpb} + I_{tpc}}{3} = \frac{(R_{LDa}R_{LDb} - R_{LDa}R_{LDc})V_C + (R_{LDa}R_{LDc} - R_{LDc}R_{LDb})V_B + (R_{LDc}R_{LDb} - R_{LDa}R_{LDb})V_A}{3R_{LDa}R_{LDb}R_{LDc}n^2} \quad [3\text{-}204]$$

$$\frac{I_{tpa} + I_{tpb} + I_{tpc}}{3} = -\frac{I_{sn}}{3n} \quad [3\text{-}205]$$

Hence the common mode current in the secondary is converted by a factor of $\frac{1}{3n}$ into a circulating current in the primary delta connected windings.

If we further simplify and assume all of the load resistances are equal to R_{LD} and the sources are balanced such that $V_A + V_B + V_C = 0$ then

$$I_{pa} = \frac{3V_A}{R_{LD}n} \quad [3\text{-}1b]$$

$$I_{pb} = \frac{3V_B}{R_{LD}n} \quad [3\text{-}2b]$$

$$I_{pc} = \frac{3V_C}{R_{LD}n} \quad [3\text{-}3b]$$

$$I_{tpa} = \frac{V_A - V_B}{R_{LD}n^2} \quad [3\text{-}4b]$$

$$I_{tpb} = \frac{V_B - V_C}{R_{LD}n^2} \quad [3\text{-}5b]$$

$$I_{tpc} = \frac{V_C - V_A}{R_{LD}n^2} \quad [3\text{-}6b]$$

$$I_{sa} = -\frac{(V_A - V_B)}{R_{LD}n} \quad [3\text{-}13b]$$

$$I_{sb} = -\frac{(V_B - V_C)}{R_{LD}n} \quad [3\text{-}14b]$$

$$I_{sc} = -\frac{(V_C - V_A)}{R_{LD}n} \quad [3\text{-}15b]$$

$$I_{tsa} = -\frac{(V_A - V_B)}{R_{LD}n} \quad [3\text{-}16b]$$

$$I_{tsb} = -\frac{(V_B - V_C)}{R_{LD}n} \qquad [3\text{-}17b]$$

$$I_{tsc} = -\frac{(V_C - V_A)}{R_{LD}n} \qquad [3\text{-}18b]$$

$$I_{sn} = 0 \qquad [3\text{-}28b]$$

$$I_{sLDa} = \frac{(V_A - V_B)}{R_{LD}n} \qquad [3\text{-}29b]$$

$$I_{sLDb} = \frac{(V_B - V_C)}{R_{LD}n} \qquad [3\text{-}30b]$$

$$I_{sLDc} = \frac{(V_C - V_A)}{R_{LD}n} \qquad [3\text{-}31b]$$

With a balanced load, the CM current in the secondary is 0 as is the circulating current in the primary.

Figure 4 depicts a delta-wye transformer with a balanced load and a ground fault on phase a of the secondary. The following equations are modified:

$$V_{sag} - V_{sLDn} - I_{sLDa}R_{LD} = 0 \qquad [3\text{-}124c]$$

$$V_{sbg} - V_{sLDn} - I_{sLDb}R_{LD} = 0 \qquad [3\text{-}125c]$$

$$V_{scg} - V_{sLDn} - I_{sLDc}R_{LD} = 0 \qquad [3\text{-}126c]$$

$$I_{sa} + I_{sLDa} + I_{sfault} = 0 \qquad [3\text{-}130c]$$

$$V_{sag} = 0 \qquad [3\text{-}134c]$$

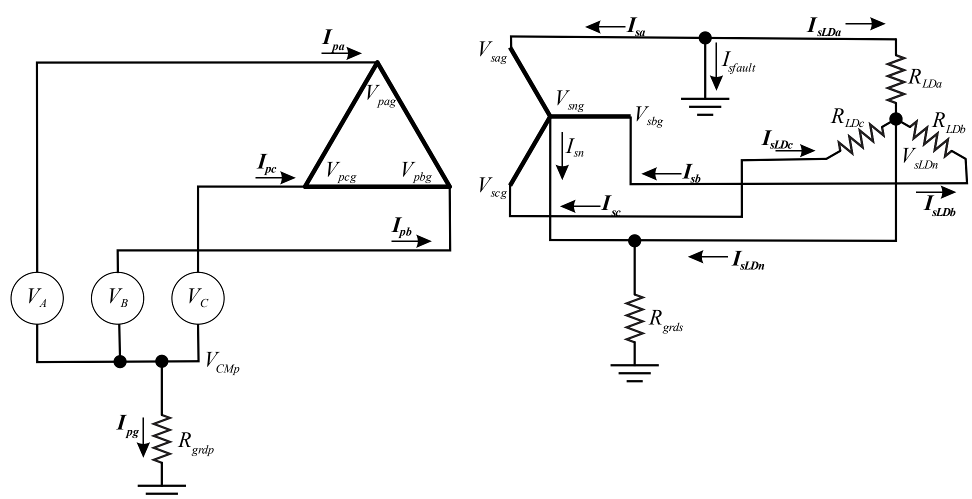


Figure 4: Delta-wye transformer with ground fault

$$I_{pa} = \frac{(-R_{LD}R_{grds}V_C + (-R_{LD}R_{grds} - R_{LD}{}^2)V_B + (2R_{LD}RR_{grds} + R_{LD}{}^2)V_A)n^2 + R_T((-R_{grds} - R_{LD})V_C + (-R_{grds} - R_{LD})V_B + (2R_{grds} + 2R_{LD})V_A)}{R_{LD}{}^2 R_{grds}n^4 + R_T(2R_{LD}R_{grds} + R_{LD}{}^2)n^2 + R_T{}^2(R_{grds} + R_{LD})} \qquad [3\text{-}1c]$$

$$I_{pb} = -\frac{(R_{LD}R_{grds}V_C+(-2R_{LD}R_{grds}-R_{LD}{}^2)V_B+(R_{LD}R_{grds}+R_{LD}{}^2)V_A)n^2+R_T((R_{grds}+R_{LD})V_C+(-2R_{grds}-2R_{LD})V_B+(R_{grds}+R_{LD})V_A)}{R_{LD}{}^2R_{grds}n^4+R_T(2R_{LD}R_{grds}+R_{LD}{}^2)n^2+R_T{}^2(R_{grds}+R_{LD})} \quad [3\text{-}2c]$$

$$I_{pc} = -\frac{-2V_C+V_B+V_A}{R_{LD}n^2+R_T} \quad [3\text{-}3c]$$

$$I_{tpa} = \frac{(-R_{grds}-R_{LD})V_B+(R_{grds}+R_{LD})V_A}{R_{LD}R_{grds}n^2+R_T(R_{grds}+R_{LD})} \quad [3\text{-}4c]$$

$$I_{tpb} = \frac{V_B-V_C}{R_{LD}n^2+R_T} \quad [3\text{-}5c]$$

$$I_{tpc} = \frac{V_C-V_A}{R_{LD}n^2+R_T} \quad [3\text{-}6c]$$

$$V_{pag} = V_A \quad [3\text{-}7c]$$

$$V_{pbg} = V_B \quad [3\text{-}8c]$$

$$V_{pcg} = V_C \quad [3\text{-}9c]$$

$$V_{tpa} = V_A - V_B \quad [3\text{-}10c]$$

$$V_{tpb} = V_B - V_C \quad [3\text{-}11c]$$

$$V_{tpc} = V_C - V_A \quad [3\text{-}12c]$$

$$I_{sa} = -\frac{((-R_{grds}-R_{LD})V_B+(R_{grds}+R_{LD})V_A)n}{R_{LD}R_{grds}n^2+R_T(R_{grds}+R_{LD})} \quad [3\text{-}13c]$$

$$I_{sb} = -\frac{(V_B-V_C)n}{R_{LD}n^2+R_T} \quad [3\text{-}14c]$$

$$I_{sc} = \frac{(V_A-V_C)n}{R_{LD}n^2+R_T} \quad [3\text{-}15c]$$

$$I_{tsa} = -\frac{((-R_{grds}-R_{LD})V_B+(R_{grds}+R_{LD})V_A)n}{R_{LD}R_{grds}n^2+R_T(R_{grds}+R_{LD})} \quad [3\text{-}16c]$$

$$I_{tsb} = -\frac{(V_B-V_C)n}{R_{LD}n^2+R_T} \quad [3\text{-}17c]$$

$$I_{tsc} = \frac{(V_A-V_C)n}{R_{LD}n^2+R_T} \quad [3\text{-}18c]$$

$$V_{sag} = 0 \quad [3\text{-}19c]$$

$$V_{sbg} = -\frac{(R_{LD}{}^2R_{grds}V_C-2R_{LD}{}^2R_{grds}V_B+R_{LD}{}^2R_{grds}V_A)n^3+R_T((R_{LD}R_{grds}+R_{LD}{}^2)V_C+(-2R_{LD}R_{grds}-R_{LD}{}^2)V_B+R_{LD}R_{grds}V_A)n}{R_{LD}{}^2R_{grds}n^4+R_T(2R_{LD}R_{grds}+R_{LD}{}^2)n^2+R_T{}^2(R_{grds}+R_{LD})} \quad [3\text{-}20c]$$

$$V_{scg} = -\frac{(-R_{LD}{}^2R_{grds}V_C-R_{LD}{}^2R_{grds}V_B+2R_{LD}{}^2R_{grds}V_A)n^3+R_T((-R_{LD}R_{grds}-R_{LD}{}^2)V_C-R_{LD}R_{grds}V_B+(2R_{LD}R_{grds}+R_{LD}{}^2)V_A)n}{R_{LD}{}^2R_{grds}n^4+R_T(2R_{LD}R_{grds}+R_{LD}{}^2)n^2+R_T{}^2(R_{grds}+R_{LD})} \quad [3\text{-}21c]$$

$$V_{tsa} = \frac{(R_{LD}R_{grds}V_A-R_{LD}R_{grds}V_B)n}{R_{LD}R_{grds}n^2+R_T(R_{grds}+R_{LD})} \quad [3\text{-}22c]$$

$$V_{tsb} = \frac{(R_{LD}V_B-R_{LD}V_C)n}{R_{LD}n^2+R_T} \quad [3\text{-}23c]$$

$$V_{tsc} = -\frac{(R_{LD}V_A-R_{LD}V_C)n}{R_{LD}n^2+R_T} \quad [3\text{-}24c]$$

$$V_{CMp} = 0 \qquad [3\text{-}25c]$$

$$I_{pg} = 0 \qquad [3\text{-}26c]$$

$$V_{sng} = -\frac{(R_{LD}R_{grds}V_A - R_{LD}R_{grds}V_B)n}{R_{LD}R_{grds}n^2 + R_T(R_{grds}+R_{LD})} \qquad [3\text{-}27c]$$

$$I_{sn} = -\frac{(R_{LD}{}^2V_A - R_{LD}{}^2V_B)n^3}{R_{LD}{}^2R_{grds}n^4 + R_T(2R_{LD}R_{grds}+R_{LD}{}^2)n^2 + R_T{}^2(R_{grds}+R_{LD})} \qquad [3\text{-}28c]$$

$$I_{sLDa} = \frac{(R_{grds}V_A - R_{grds}V_B)n}{R_{LD}R_{grds}n^2 + R_T(R_{grds}+R_{LD})} \qquad [3\text{-}29c]$$

$$I_{sLDb} = \frac{(V_B - V_C)n}{R_{LD}n^2 + R_T} \qquad [3\text{-}30c]$$

$$I_{sLDc} = -\frac{(V_A - V_C)n}{R_{LD}n^2 + R_T} \qquad [3\text{-}31c]$$

$$I_{sLDn} = -\frac{RT(R_{LD}V_A - R_{LD}V_B)n}{R_{LD}{}^2R_{grds}n^4 + R_T(2R_{LD}R_{grds}+R_{LD}{}^2)n^2 + R_T{}^2(R_{grds}+R_{LD})} \qquad [3\text{-}32c]$$

$$V_{sLDn} = -\frac{(R_{LD}R_{grds}V_A - R_{LD}R_{grds}V_B)n}{R_{LD}R_{grds}n^2 + R_T(R_{grds}+R_{LD})} \qquad [3\text{-}33c]$$

$$I_{sfault} = \frac{(R_{LD}V_A - R_{LD}V_B)n}{R_{LD}R_{grds}n^2 + R_T(R_{grds}+R_{LD})} \qquad [3\text{-}34c]$$

If we also assume R_T is negligible, then

$$I_{pa} = \frac{-R_{LD}V_B + (3R_{grds}+R_{LD})V_A}{R_{LD}R_{grds}n^2} \qquad [3\text{-}1d]$$

$$I_{pb} = \frac{(3R_{grds}+R_{LD})V_B - R_{LD}V_A}{R_{LD}R_{grds}n^2} \qquad [3\text{-}2d]$$

$$I_{pc} = \frac{3V_C}{R_{LD}n^2} \qquad [3\text{-}3d]$$

$$I_{tpa} = \frac{(R_{grds}+R_{LD})V_A - (R_{grds}+R_{LD})V_B}{R_{LD}R_{grds}n^2} \qquad [3\text{-}4d]$$

$$I_{tpb} = \frac{V_B - V_C}{R_{LD}n^2} \qquad [3\text{-}5d]$$

$$I_{tpc} = \frac{V_C - V_A}{R_{LD}n^2} \qquad [3\text{-}6d]$$

$$I_{sa} = \frac{((R_{grds}+R_{LD})V_B - (R_{grds}+R_{LD})V_A)}{R_{LD}R_{grds}n} \qquad [3\text{-}13d]$$

$$I_{sb} = \frac{(V_C - V_B)}{R_{LD}n} \qquad [3\text{-}14d]$$

$$I_{sc} = \frac{(V_A - V_C)}{R_{LD}n} \qquad [3\text{-}15d]$$

$$I_{tsa} = \frac{((R_{grds}+R_{LD})V_B - (R_{grds}+R_{LD})V_A)}{R_{LD}R_{grds}n} \qquad [3\text{-}16d]$$

$$I_{tsb} = \frac{(V_C - V_B)}{R_{LD} n}$$ [3-17d]

$$I_{tsc} = \frac{(V_A - V_C)}{R_{LD} n}$$ [3-18d]

$$V_{sag} = 0$$ [3-19d]

$$V_{sbg} = \frac{3V_B}{n}$$ [3-20d]

$$V_{scg} = -\frac{3V_A}{n}$$ [3-21d]

$$V_{tsa} = \frac{(V_A - V_B)}{n}$$ [3-22d]

$$V_{tsb} = \frac{(V_B - V_C)}{n}$$ [3-23d]

$$V_{tsc} = \frac{(V_C - V_A)}{n}$$ [3-24d]

$$V_{CMp} = 0$$ [3-25d]

$$I_{pg} = 0$$ [3-26c]

$$V_{sng} = \frac{(V_B - V_A)}{n}$$ [3-27d]

$$I_{sn} = \frac{(V_B - V_A)}{R_{grds} n}$$ [3-28d]

$$I_{sLDa} = \frac{(V_A - V_B)}{R_{LD} n}$$ [3-29d]

$$I_{sLDb} = \frac{(V_B - V_C)}{R_{LD} n}$$ [3-30d]

$$I_{sLDc} = \frac{(V_C - V_A)}{R_{LD} n}$$ [3-31d]

$$I_{sLDn} = 0$$ [3-32d]

$$V_{sLDn} = \frac{(V_B - V_A)}{n}$$ [3-33d]

$$I_{sfault} = \frac{(V_A - V_B)}{R_{grds} n}$$ [3-34d]

Conclusions from this analysis includes:

(a) Ground fault on the secondary side results in the phase currents on the primary side not being exactly 120˚ apart
(b) The load currents are not sensitive to the ground fault.
(c) The average of the currents of the primary windings are

$$\frac{1}{3}\left(I_{tpa} + I_{tpb} + I_{tpc}\right) = \frac{1}{3}\left(\frac{(R_{grds}+R_{LD})V_A - (R_{grds}+R_{LD})V_B}{R_{LD} R_{grds} n^2} + \frac{V_B - V_C}{R_{LD} n^2} + \frac{V_C - V_A}{R_{LD} n^2}\right)$$ [3-201d]

$$\frac{1}{3}\left(I_{tpa} + I_{tpb} + I_{tpc}\right) = \frac{1}{3}\left(\frac{V_A - V_B}{R_{grds} n^2}\right) = \frac{I_{sfault}}{3n}$$ [3-202d]

Thus, the fault current on the secondary side is transformed with a factor of $\frac{1}{3n}$ into a circulating current on the primary delta winding.

If the secondary is solidly grounded, the fault current is limited only by R_T and the impedance of cables and the hull.

4. Delta-Delta Transformers

Three phase delta-delta transformers are typically used in shipboard power systems where the secondary is part of an ungrounded distribution system. This document explores the steady-state performance of three "nearly ideal" transformers when connected into a delta-delta transformer bank. Each transformer is assumed to be an ideal transformer with each transformer winding having a series resistance. If this resistance is assumed to have a complex value, it can represent a general impedance. The delta-delta transformer configurations are provided with ideal voltage sources and with resistive loads. The resistive load value may also be complex. Cases with unbalanced load resistances and with ground faults are analyzed.

Figure 5 depicts the schematic for a delta-delta connected transformer as well as the typical schematic symbol used in many power system drawings. In the schematic symbol, thick parallel lines represent the primary and secondary windings of an equivalent (or actual) single phase transformer. As shown in Chapter 2, the resistances in the primary (R_p) and secondary (R_s) can be combined into a total resistance (R_T) where n is the turns ratio of the single phase transformers.

$$R_T = R_p + R_s\, n^2 \qquad [2\text{-}107]$$

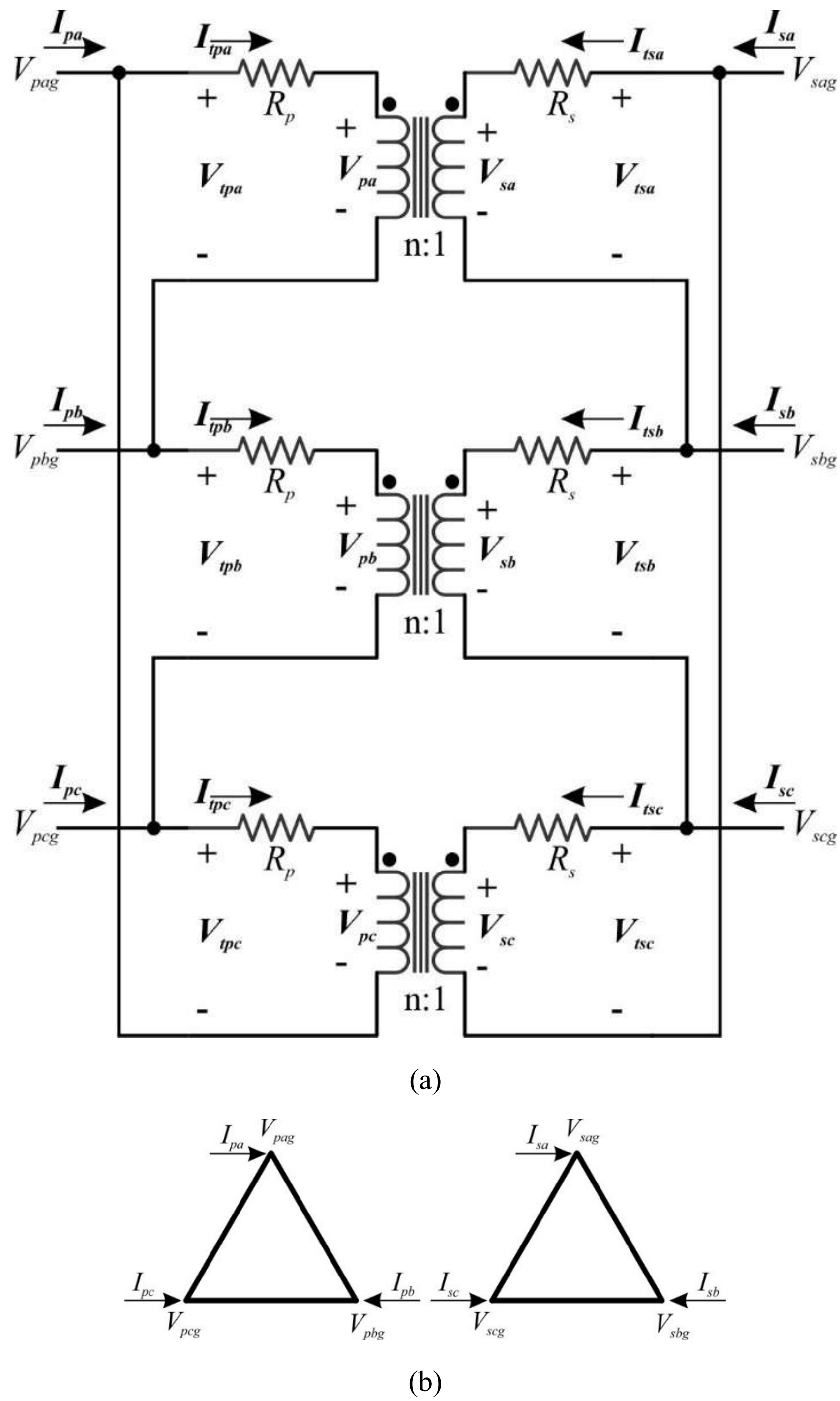


Figure 5: Delta-delta transformer – (a) circuit diagram (b) schematic.

The voltages and currents of the primary and secondary windings are related.

$$I_{pa} + I_{tpc} - I_{tpa} = 0 \quad [4\text{-}101]$$

$$I_{pb} + I_{tpa} - I_{tpb} = 0 \quad [4\text{-}102]$$

$$I_{pc} + I_{tpb} - I_{tpc} = 0 \quad [4\text{-}103]$$

$$nI_{tpa} + I_{tsa} = 0 \quad [4\text{-}104]$$

$$nI_{tpb} + I_{tsb} = 0 \quad [4\text{-}105]$$

$$nI_{tpc} + I_{tsc} = 0 \quad [4\text{-}106]$$

$$R_T I_{tpa} - V_{tpa} + nV_{tsa} = 0 \quad [4\text{-}107]$$

$$R_T I_{tpb} - V_{tpb} + nV_{tsb} = 0 \quad [4\text{-}108]$$

$$R_T I_{tpc} - V_{tpc} + nV_{tsc} = 0 \quad [4\text{-}109]$$

$$V_{tpa} - V_{pag} + V_{pbg} = 0 \quad [4\text{-}110]$$

$$V_{tpb} - V_{pbg} + V_{pcg} = 0 \quad [4\text{-}111]$$

$$V_{tpc} - V_{pcg} + V_{pag} = 0 \quad [4\text{-}112]$$

$$I_{sa} + I_{tsc} - I_{tsa} = 0 \quad [4\text{-}113]$$

$$I_{sb} + I_{tsa} - I_{tsb} = 0 \quad [4\text{-}114]$$

$$I_{sc} + I_{tsb} - I_{tsc} = 0 \quad [4\text{-}115]$$

$$V_{tsa} - V_{sag} + V_{sbg} = 0 \quad [4\text{-}116]$$

$$V_{tsb} - V_{sbg} + V_{scg} = 0 \quad [4\text{-}117]$$

$$V_{tsc} - V_{scg} + V_{sag} = 0 \quad [4\text{-}118]$$

While the equations above describe the relationships among the transformer terminals, the following equations are included to fully define the circuit shown in Figure 6. This figure includes a three-phase source with the neutral grounded with a resistor. It also includes a 3-wire load on the secondary system in addition to parasitic line to ground impedances. Each of the load resistances may be different.

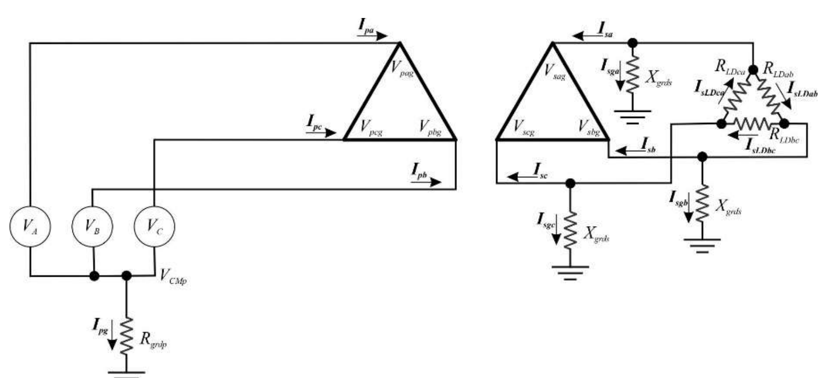


Figure 6: Delta-delta transformer with source and load

Assume a three-phase source (V_A, V_B and V_C are assumed to be sinusoidal and of the same frequency, but the equations and solution below only requires them to be ac)

$$V_{pag} - V_{CMp} = V_A \quad [4\text{-}119]$$

$$V_{pbg} - V_{CMp} = V_B \quad [4\text{-}120]$$

$$V_{pcg} - V_{CMp} = V_C \quad [4\text{-}121]$$

Assume a grounding resistor at the source

$$R_{grdp} I_{pg} - V_{CMp} = 0 \quad [4\text{-}122]$$

$$I_{pg} + I_{pa} + I_{pb} + I_{pc} = 0 \quad [4\text{-}123]$$

Assume a delta connected load where each of the load resistances may be different

$$V_{sag} - V_{sbg} - I_{sLDab} R_{LDab} = 0 \quad [4\text{-}124]$$

$$V_{sbg} - V_{scg} - I_{sLDbc} R_{LDbc} = 0 \quad [4\text{-}125]$$

$$V_{scg} - V_{sag} - I_{sLDca} R_{LDca} = 0 \quad [4\text{-}126]$$

Account for the line to ground parasitic impedance

$$V_{sag} - I_{sga} X_{grds} = 0 \quad [4\text{-}127]$$

$$V_{sbg} - I_{sgb} X_{grds} = 0 \quad [4\text{-}128]$$

$$V_{scg} - I_{sgc} X_{grds} = 0 \quad [4\text{-}129]$$

Relate the secondary currents

$$I_{sa} + I_{sga} + I_{sLDab} - I_{sLDca} = 0 \quad [4\text{-}130]$$

$$I_{sb} + I_{sgb} + I_{sLDbc} - I_{sLDab} = 0 \quad [4\text{-}131]$$

$$I_{sc} + I_{sgc} + I_{sLDca} - I_{sLDbc} = 0 \quad [4\text{-}132]$$

Solving this set of equations results in very complex expressions that are hard to understand. If we make the assumption that the load resistances are all equal to R_{LD}, then

$$I_{pa} = \frac{V_A(6X_{grds}+2R_{LD}) - V_C(3X_{grds}+R_{LD}) - V_B(3X_{grds}+R_{LD})}{3R_{LD}X_{grds}n^2 + R_T(3X_{grds}+R_{LD})} \quad [4\text{-}1a]$$

$$I_{pb} = \frac{-V_C(3X_{grds}+R_{LD}) - V_A(3X_{grds}+R_{LD}) + V_B(6X_{grds}+2R_{LD})}{3R_{LD}X_{grds}n^2 + R_T(3X_{grds}+R_{LD})} \quad [4\text{-}2a]$$

$$I_{pc} = \frac{-V_B(3X_{grds}+R_{LD}) - V_A(3X_{grds}+R_{LD}) + V_C(6X_{grds}+2R_{LD})}{3R_{LD}X_{grds}n^2 + R_T(3X_{grds}+R_{LD})} \quad [4\text{-}3a]$$

$$I_{tpa} = \frac{V_A(3X_{grds}+R_{LD}) - V_B(3X_{grds}+R_{LD})}{3R_{LD}X_{grds}n^2 + R_T(3X_{grds}+R_{LD})} \quad [4\text{-}4a]$$

$$I_{tpb} = \frac{V_B(3X_{grds}+R_{LD}) - V_C(3X_{grds}+R_{LD})}{3R_{LD}X_{grds}n^2 + R_T(3X_{grds}+R_{LD})} \quad [4\text{-}5a]$$

$$I_{tpc} = -\frac{V_A(3X_{grds}+R_{LD}) - V_C(3X_{grds}+R_{LD})}{3R_{LD}X_{grds}n^2 + R_T(3X_{grds}+R_{LD})} \quad [4\text{-}6a]$$

$$V_{pag} = V_A \quad [4\text{-}7a]$$

$$V_{pbg} = V_B \quad [4\text{-}8a]$$

$$V_{pcg} = V_C \quad [4\text{-}9a]$$

$$V_{tpa} = V_A - V_B \quad [4\text{-}10a]$$

$$V_{tpb} = V_B - V_C \quad [4\text{-}11a]$$

$$V_{tpc} = V_C - V_A \quad [4\text{-}12a]$$

$$I_{sa} = \frac{(-V_A(6X_{grds}+2R_{LD})+V_C(3X_{grds}+R_{LD})+V_B(3X_{grds}+R_{LD}))n}{3R_{LD}X_{grds}n^2+R_T(3X_{grds}+R_{LD})} \quad [4\text{-}13a]$$

$$I_{sb} = \frac{(V_C(3X_{grds}+R_{LD})+V_A(3X_{grds}+R_{LD})-V_B(6X_{grds}+2R_{LD}))n}{3R_{LD}X_{grds}n^2+R_T(3X_{grds}+R_{LD})} \quad [4\text{-}14a]$$

$$I_{sc} = \frac{(V_B(3X_{grds}+R_{LD})+V_A(3X_{grds}+R_{LD})-V_C(6X_{grds}+2R_{LD}))n}{3R_{LD}X_{grds}n^2+R_T(3X_{grds}+R_{LD})} \quad [4\text{-}15a]$$

$$I_{tsa} = \frac{(-V_A+V_B)(3X_{grds}+R_{LD})n}{3R_{LD}X_{grds}n^2+R_T(3X_{grds}+R_{LD})} \quad [4\text{-}16a]$$

$$I_{tsb} = \frac{(-V_B+V_C)(3X_{grds}+R_{LD})n}{3R_{LD}X_{grds}n^2+R_T(3X_{grds}+R_{LD})} \quad [4\text{-}17a]$$

$$I_{tsc} = \frac{(V_A-V_C)(3X_{grds}+R_{LD})n}{3R_{LD}X_{grds}n^2+R_T(3X_{grds}+R_{LD})} \quad [4\text{-}18a]$$

$$V_{sag} = \frac{(-R_{LD}V_CX_{grds}-R_{LD}V_BX_{grds}+2R_{LD}V_AX_{grds})n}{3R_{LD}X_{grds}n^2+R_T(3X_{grds}+R_{LD})} \quad [4\text{-}19a]$$

$$V_{sbg} = \frac{(-R_{LD}V_CX_{grds}+2R_{LD}V_BX_{grds}-R_{LD}V_AX_{grds})n}{3R_{LD}X_{grds}n^2+R_T(3X_{grds}+R_{LD})} \quad [4\text{-}20a]$$

$$V_{scg} = \frac{(2R_{LD}V_CX_{grds}-R_{LD}V_BX_{grds}-R_{LD}V_AX_{grds})n}{3R_{LD}X_{grds}n^2+R_T(3X_{grds}+R_{LD})} \quad [4\text{-}21a]$$

$$V_{tsa} = \frac{(3R_{LD}V_AX_{grds}-3R_{LD}V_BX_{grds})n}{3R_{LD}X_{grds}n^2+R_T(3X_{grds}+R_{LD})} \quad [4\text{-}22a]$$

$$V_{tsb} = \frac{(3R_{LD}V_BX_{grds}-3R_{LD}V_CX_{grds})n}{3R_{LD}X_{grds}n^2+R_T(3X_{grds}+R_{LD})} \quad [4\text{-}23a]$$

$$V_{tsc} = \frac{(-3R_{LD}V_AX_{grds}+3R_{LD}V_CX_{grds})n}{3R_{LD}X_{grds}n^2+R_T(3X_{grds}+R_{LD})} \quad [4\text{-}24a]$$

$$V_{CMp} = 0 \quad [4\text{-}25a]$$

$$I_{pg} = 0 \quad [4\text{-}26a]$$

$$I_{sLDab} = \frac{(3V_AX_{grds}-3V_BX_{grds})n}{3R_{LD}X_{grds}n^2+R_T(3X_{grds}+R_{LD})} \quad [4\text{-}27a]$$

$$I_{sLDbc} = \frac{(3V_BX_{grds}-3V_CX_{grds})n}{3R_{LD}X_{grds}n^2+R_T(3X_{grds}+R_{LD})} \quad [4\text{-}28a]$$

$$I_{sLDca} = \frac{(-3V_A X_{grds} + 3V_C X_{grds})n}{3R_{LD}X_{grds}n^2 + R_T(3X_{grds} + R_{LD})} \quad [4\text{-}29a]$$

$$I_{sga} = -\frac{(R_{LD}V_C + R_{LD}V_B - 2R_{LD}V_A)n}{3R_{LD}X_{grds}n^2 + R_T(3X_{grds} + R_{LD})} \quad [4\text{-}30a]$$

$$I_{sgb} = -\frac{(R_{LD}V_C - 2R_{LD}V_B + R_{LD}V_A)n}{3R_{LD}X_{grds}n^2 + R_T(3X_{grds} + R_{LD})} \quad [4\text{-}31a]$$

$$I_{sgc} = -\frac{(-2R_{LD}V_C + R_{LD}V_B + R_{LD}V_A)n}{3R_{LD}X_{grds}n^2 + R_T(3X_{grds} + R_{LD})} \quad [4\text{-}32a]$$

If we further make the assumption that the sources are balanced and 120˚ apart, then $V_A + V_B + V_C = 0$ and several of the solutions may be simplified:

$$I_{pa} = \frac{3V_A(3X_{grds} + R_{LD})}{3R_{LD}X_{grds}n^2 + R_T(3X_{grds} + R_{LD})} \quad [4\text{-}1b]$$

$$I_{pb} = \frac{3V_B(3X_{grds} + R_{LD})}{3R_{LD}X_{grds}n^2 + R_T(3X_{grds} + R_{LD})} \quad [4\text{-}2b]$$

$$I_{pc} = \frac{3V_C(3X_{grds} + R_{LD})}{3R_{LD}X_{grds}n^2 + R_T(3X_{grds} + R_{LD})} \quad [4\text{-}3b]$$

$$I_{sa} = \frac{-3V_A(3X_{grds} + R_{LD})n}{3R_{LD}X_{grds}n^2 + R_T(3X_{grds} + R_{LD})} \quad [4\text{-}13b]$$

$$I_{sb} = \frac{-3V_B(3X_{grds} + R_{LD})n}{3R_{LD}X_{grds}n^2 + R_T(3X_{grds} + R_{LD})} \quad [4\text{-}14b]$$

$$I_{sc} = \frac{-3V_C(3X_{grds} + R_{LD})n}{3R_{LD}X_{grds}n^2 + R_T(3X_{grds} + R_{LD})} \quad [4\text{-}15b]$$

$$V_{sag} = \frac{(3R_{LD}V_A X_{grds})n}{3R_{LD}X_{grds}n^2 + R_T(3X_{grds} + R_{LD})} \quad [4\text{-}19b]$$

$$V_{sbg} = \frac{(3R_{LD}V_B X_{grds})n}{3R_{LD}X_{grds}n^2 + R_T(3X_{grds} + R_{LD})} \quad [4\text{-}20b]$$

$$V_{scg} = \frac{(3R_{LD}V_C X_{grds})n}{3R_{LD}X_{grds}n^2 + R_T(3X_{grds} + R_{LD})} \quad [4\text{-}21b]$$

$$I_{sga} = \frac{(3R_{LD}V_A)n}{3R_{LD}X_{grds}n^2 + R_T(3X_{grds} + R_{LD})} \quad [4\text{-}30b]$$

$$I_{sgb} = \frac{(3R_{LD}V_B)n}{3R_{LD}X_{grds}n^2 + R_T(3X_{grds} + R_{LD})} \quad [4\text{-}31b]$$

$$I_{sgc} = \frac{(3R_{LD}V_C)n}{3R_{LD}X_{grds}n^2 + R_T(3X_{grds} + R_{LD})} \quad [4\text{-}32b]$$

With balanced sources and loads, the phase currents are also balanced since each is equally proportional to their corresponding source voltage.

If we further make the assumptions that X_{grds} is very large and R_T is very small, then the familiar ideal transformer relationships are derived:

$$I_{pa} = \frac{3V_A}{R_{LD}n^2} \quad [4\text{-}1c]$$

$$I_{pb} = \frac{3V_B}{R_{LD}n^2} \qquad [4\text{-}2c]$$

$$I_{pc} = \frac{3V_C}{R_{LD}n^2} \qquad [4\text{-}3c]$$

$$I_{sa} = \frac{-3V_A}{R_{LD}n} \qquad [4\text{-}13c]$$

$$I_{sb} = \frac{-3V_B}{R_{LD}n} \qquad [4\text{-}14c]$$

$$I_{sc} = \frac{-3V_C}{R_{LD}n} \qquad [4\text{-}15c]$$

$$V_{sag} = \frac{V_A}{n} \qquad [4\text{-}19c]$$

$$V_{sbg} = \frac{V_B}{n} \qquad [4\text{-}20c]$$

$$V_{scg} = \frac{V_C}{n} \qquad [4\text{-}21c]$$

$$I_{sga} = 0 \qquad [4\text{-}30c]$$

$$I_{sgb} = 0 \qquad [4\text{-}31c]$$

$$I_{sgc} = 0 \qquad [4\text{-}32c]$$

If we assume the loads are not balanced, but the sources are balanced, X_{grds} is large and R_T is negligible, then:

$$I_{pa} = \frac{(V_A(R_{LDab}+R_{LDca})-V_BR_{LDca}-V_CR_{LDab})}{R_{LDab}R_{LDca}n^2} \qquad [4\text{-}1d]$$

$$I_{pb} = \frac{(V_B(R_{LDab}+R_{LDbc})-V_AR_{LDbc}-V_CR_{LDab})}{R_{LDab}R_{LDbc}n^2} \qquad [4\text{-}2d]$$

$$I_{pc} = \frac{(V_C(R_{LDbc}+R_{LDca})-V_BR_{LDca}-V_AR_{LDbc})}{R_{LDbc}R_{LDca}n^2} \qquad [4\text{-}3d]$$

$$I_{sa} = \frac{(-V_A(R_{LDab}+R_{LDca})+V_BR_{LDca}+V_CR_{LDab})}{R_{LDab}R_{LDca}n} \qquad [4\text{-}13d]$$

$$I_{sb} = \frac{(-V_B(R_{LDab}+R_{LDbc})+V_AR_{LDbc}+V_CR_{LDab})}{R_{LDab}R_{LDbc}n} \qquad [4\text{-}14d]$$

$$I_{sc} = \frac{(-V_C(R_{LDbc}+R_{LDca})+V_BR_{LDca}+V_AR_{LDbc})}{R_{LDbc}R_{LDca}n} \qquad [4\text{-}15d]$$

$$V_{sag} = \frac{V_A}{n} \qquad [4\text{-}19d]$$

$$V_{sbg} = \frac{V_B}{n} \qquad [4\text{-}20d]$$

$$V_{scg} = \frac{V_C}{n} \qquad [4\text{-}21d]$$

Unbalanced loads may result in primary and secondary currents that are not 120˚ apart, but the voltages are not impacted by the unbalanced loads.

Figure 2-4 depicts a delta-delta transformer with a balanced load and a ground fault on phase a of the secondary. The following equations are modified:

$$V_{sag} - V_{sbg} - I_{sLDab}R_{LD} = 0 \quad [4\text{-}124a]$$

$$V_{sbg} - V_{scg} - I_{sLDbc}R_{LD} = 0 \quad [4\text{-}125a]$$

$$V_{scg} - V_{sag} - I_{sLDca}R_{LD} = 0 \quad [4\text{-}126a]$$

$$V_{sag} - I_{sga}X_{grds} = 0 \quad [4\text{-}127a]$$

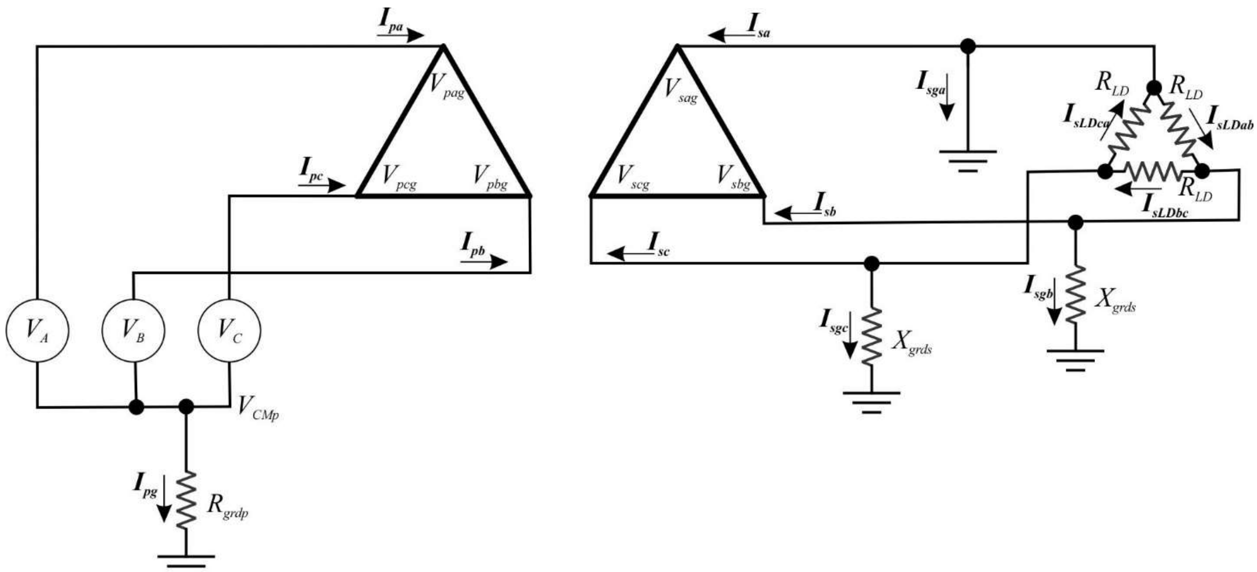


Figure 7: Delta-delta transformer with source and balanced load with ground fault

If we also assume R_T is negligible, then

$$I_{pa} = \frac{V_A(3X_{grds}+3R_{LD})}{R_{LD}X_{grds}n^2} \quad [4\text{-}1e]$$

$$I_{pb} = \frac{-V_AR_{LD}+V_B(3X_{grds}+R_{LD})}{R_{LD}X_{grds}n^2} \quad [4\text{-}2e]$$

$$I_{pc} = \frac{-V_AR_{LD}+V_C(3X_{grds}+R_{LD})}{R_{LD}X_{grds}n^2} \quad [4\text{-}3e]$$

$$I_{sa} = \frac{-V_A(3X_{grds}+3R_{LD})}{R_{LD}X_{grds}n} \quad [4\text{-}13e]$$

$$I_{sb} = \frac{V_AR_{LD}-V_B(3X_{grds}+R_{LD})}{R_{LD}X_{grds}n} \quad [4\text{-}14e]$$

$$I_{sc} = \frac{V_AR_{LD}-V_C(3X_{grds}+R_{LD})}{R_{LD}X_{grds}n} \quad [4\text{-}15e]$$

$$I_{tsa} = \frac{-V_A\left(3X_{grds}+4R_{LD}\right)+V_B\left(3X_{grds}+R_{LD}\right)}{3R_{LD}X_{grds}n} \quad [4\text{-}16e]$$

$$I_{tsb} = \frac{-V_B(3X_{grds}+R_{LD})+V_C(3X_{grds}+R_{LD})}{3R_{LD}X_{grds}n} \quad [4\text{-}17e]$$

$$I_{tsc} = \frac{V_A(3X_{grds}+4R_{LD})-V_C(3X_{grds}+R_{LD})}{3R_{LD}X_{grds}n} \quad [4\text{-}18e]$$

$$V_{sag} = 0 \quad [4\text{-}19e]$$

$$V_{sbg} = \frac{(V_B-V_A)}{n} \quad [4\text{-}20e]$$

$$V_{scg} = \frac{(V_C-V_A)}{n} \quad [4\text{-}21e]$$

$$I_{sga} = \frac{-V_C-V_B+2V_A}{X_{grds}n} \quad [4\text{-}30e]$$

$$I_{sgb} = \frac{(V_B-V_A)}{X_{grds}n} \quad [4\text{-}31e]$$

$$I_{sgc} = \frac{(V_C-V_A)}{X_{grds}n} \quad [4\text{-}32e]$$

$$I_{sLDab} = \frac{(V_A-V_B)}{R_{LD}n} \quad [4\text{-}27e]$$

$$I_{sLDbc} = \frac{(V_B-V_C)}{R_{LD}n} \quad [4\text{-}28e]$$

$$I_{sLDca} = \frac{(V_C-V_A)}{R_{LD}n} \quad [4\text{-}29e]$$

Conclusions from this analysis includes:

(a) Since the model of the delta windings does not include provisions for parasitic capacitances to ground, the sum of the currents into the primary must sum to zero, and the sum of the currents out of the secondary must sum to zero.
(b) Unbalanced loads manifest as currents that in general are not equal in magnitude and not 120˚ electrical degrees apart.
(c) Ground faults manifest as an unbalanced load with respect to the impedance to ground.
(d) Ground faults do not impact the loads.

If one of the transformers is removed from the circuit as depicted in Figure 8, the configuration is known as a broken-delta-delta transformer or a V-V transformer. Assuming a balanced load results in the following modifications to the original equations

$$I_{tpa} = 0 \quad [4\text{-}107b]$$

$$V_{sag} - V_{sbg} - I_{sLDab}R_{LD} = 0 \quad [4\text{-}124b]$$

$$V_{sbg} - V_{scg} - I_{sLDbc}R_{LD} = 0 \quad [4\text{-}125b]$$

$$V_{scg} - V_{sag} - I_{sLDca}R_{LD} = 0 \quad [4\text{-}126b]$$

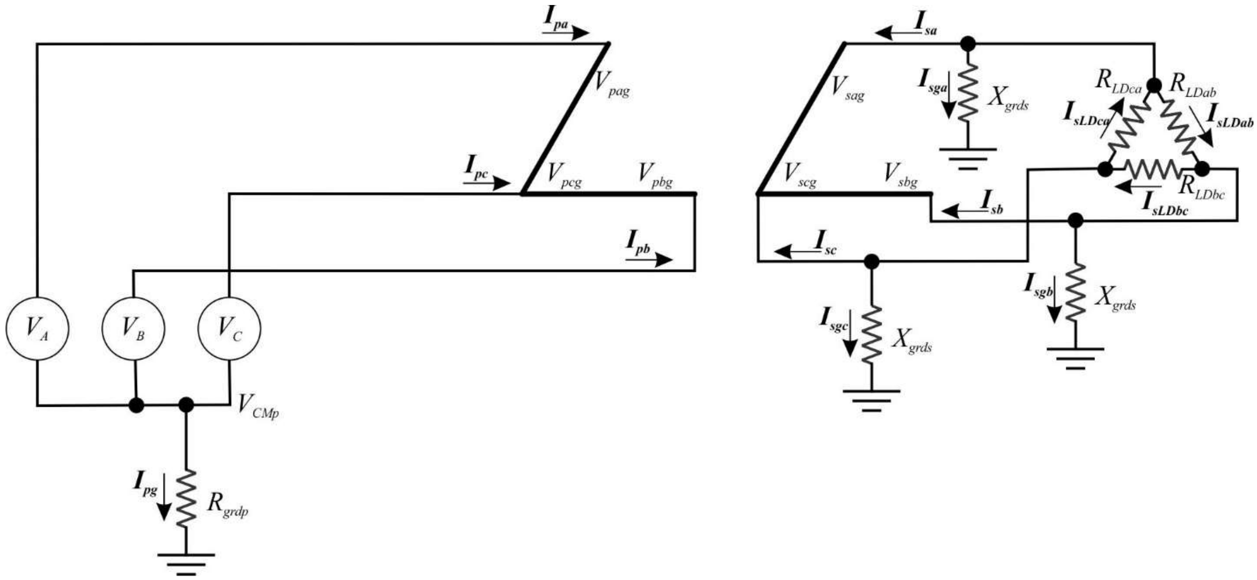


Figure 8: Delta-delta transformer with source and balanced load with ground fault

Assuming R_T is negligible results in:

$$I_{pa} = \frac{V_A\left(3X_{grds}+R_{LD}\right)}{R_{LD}X_{grds}n^2} \quad [4\text{-}1f]$$

$$I_{pb} = \frac{V_B\left(3X_{grds}+R_{LD}\right)}{R_{LD}X_{grds}n^2} \quad [4\text{-}2f]$$

$$I_{pc} = \frac{V_C\left(3X_{grds}+R_{LD}\right)}{R_{LD}X_{grds}n^2} \quad [4\text{-}3f]$$

$$I_{tpa} = 0 \quad [4\text{-}4f]$$

$$I_{tpb} = \frac{V_B\left(3X_{grds}+R_{LD}\right)}{R_{LD}X_{grds}n^2} \quad [4\text{-}5f]$$

$$I_{tpc} = -\frac{V_A(3X_{grds}+R_{LD})}{R_{LD}X_{grds}n^2} \quad [4\text{-}6f]$$

$$I_{sa} = -\frac{V_A(3X_{grds}+R_{LD})}{R_{LD}X_{grds}n} \quad [4\text{-}13f]$$

$$I_{sb} = -\frac{V_B(3X_{grds}+R_{LD}))}{R_{LD}X_{grds}n} \quad [4\text{-}14f]$$

$$I_{sc} = -\frac{V_C(3X_{grds}+R_{LD}))}{R_{LD}X_{grds}n} \quad [4\text{-}15f]$$

$$V_{sag} = \frac{V_A}{n} \quad [4\text{-}19f]$$

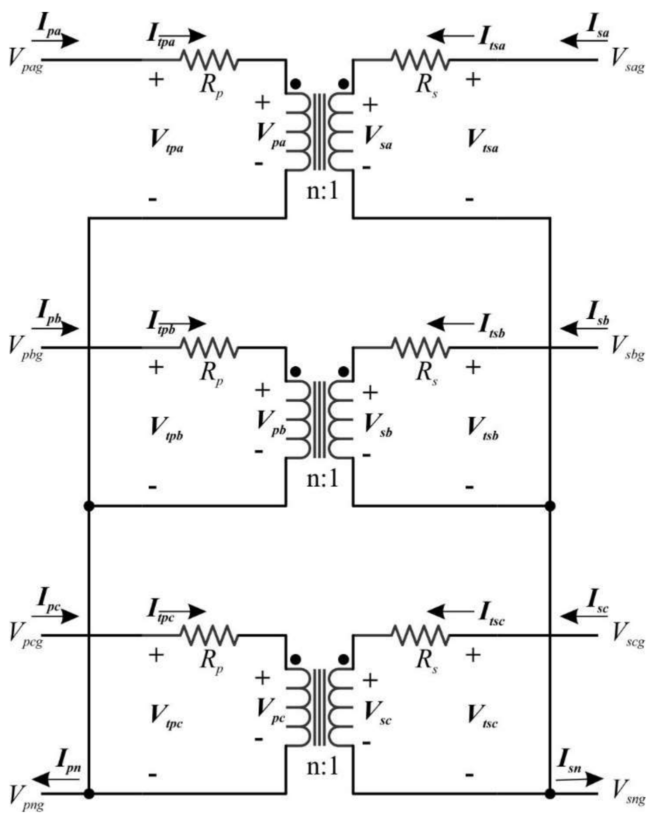
$$V_{sbg} = \frac{V_B}{n} \quad [4\text{-}20f]$$

$$V_{scg} = \frac{V_C}{n} \quad [4\text{-}21f]$$

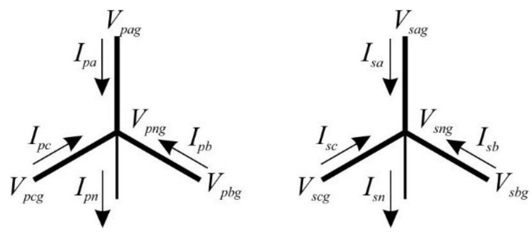
5. Wye-Wye Transformers

Three phase wye-wye transformers are not typically used in shipboard power systems. This document explores the steady-state performance of three "nearly ideal" transformers when connected into a wye-wye transformer bank. Each transformer is assumed to be an ideal transformer with each transformer winding having a series resistance. If this resistance is assumed to have a complex value, it can represent a general impedance. The wye-wye transformer configurations are provided with ideal voltage sources and with resistive loads. The resistive load value may also be complex. Various grounding schemes are also examined. Cases with unbalanced load resistances and with ground faults are analyzed.

Figure 9 depicts the schematic for a wye-wye connected transformer as well as the typical schematic symbol used in many power system drawings. In the schematic symbol, thick parallel lines represent the primary and secondary windings of an equivalent (or actual) single phase transformer. For completeness, Figure 9 depicts a neutral connection, with possible neutral current, on both the primary and secondary windings. Normally, the neutral connection on the primary winding is left unterminated with the design intention for the primary winding neutral current to be zero. In reality, there will be a small parasitic capacitance from the primary windings to ground which may be modelled as a parasitic capacitance from the neutral connection to ground.



(a)



(b)

Figure 9: Wye-wye transformer – (a) circuit diagram (b) schematic

The voltages and currents of the primary and secondary windings are related

$$I_{pa} + I_{pb} + I_{pc} - I_{pn} = 0 \qquad [5\text{-}101]$$

$$nI_{pa} + I_{sa} = 0 \qquad [5\text{-}102]$$

$$nI_{pb} + I_{sb} = 0 \qquad [5\text{-}103]$$

$$nI_{pc} + I_{sc} = 0 \qquad [5\text{-}104]$$

$$R_T I_{pa} - V_{tpa} + nV_{tsa} = 0 \qquad [5\text{-}105]$$

$$R_T I_{pb} - V_{tpb} + nV_{tsb} = 0 \qquad [5\text{-}106]$$

$$R_T I_{pc} - V_{tpc} + nV_{tsc} = 0 \qquad [5\text{-}107]$$

$$V_{tpa} - V_{pag} + V_{png} = 0 \qquad [5\text{-}108]$$

$$V_{tpb} - V_{pbg} + V_{png} = 0 \qquad [5\text{-}109]$$

$$V_{tpc} - V_{pbc} + V_{png} = 0 \qquad [5\text{-}110]$$

$$V_{tsa} - V_{sag} + V_{sng} = 0 \qquad [5\text{-}111]$$

$$V_{tsb} - V_{sbg} + V_{sng} = 0 \qquad [5\text{-}112]$$

$$V_{tsc} - V_{sbc} + V_{sng} = 0 \qquad [5\text{-}113]$$

$$I_{sa} + I_{sb} + I_{sc} - I_{sn} = 0 \qquad [5\text{-}114]$$

While the equations above describe the relationships among the transformer terminals, the following equations are included to fully define the circuit shown in Figure 10. This figure includes a three-phase source with the neutral grounded with a resistor. It also includes a 4-wire load on the secondary system in addition to a grounding resistor connected to the secondary winding neutral conductor. Each of the load resistances may be different.

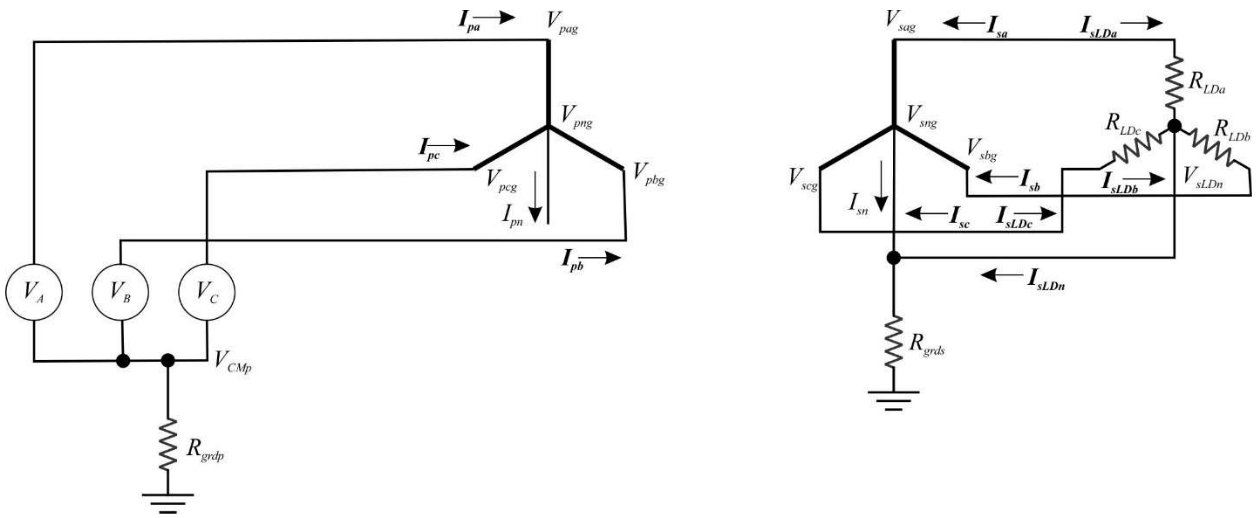


Figure 10: Wye-wye transformer with source and load.

Assume a three phase source (V_A, V_B and V_C are assumed to be sinusoidal and of the same frequency, but the equations and solution below only requires them to be ac)

$$V_{pag} - V_{CMp} = V_A \quad [5\text{-}115]$$

$$V_{pbg} - V_{CMp} = V_B \quad [5\text{-}116]$$

$$V_{pcg} - V_{CMp} = V_C \quad [5\text{-}117]$$

Assume a grounding resistor at the source and the CM current from the transformer primary returns here

$$R_{grdp} I_{pn} + V_{CMp} = 0 \quad [5\text{-}118]$$

Assume the neutral connection at the transformer primary is unterminated

$$I_{pn} = 0 \quad [5\text{-}119]$$

Assume a 4 wire system on the secondary with a grounding resistor. I_{sLDn} is the neutral connection of the 4 wire load back to the neutral connection of the secondary.

$$R_{grds} I_{sn} - V_{sng} + R_{grds} I_{sLDn} = 0 \quad [5\text{-}120]$$

Assume a resistive load where each of the phase resistances can be different

$$\frac{1}{R_{LD}} V_{sag} - I_{sLDa} - \frac{1}{R_{LDa}} V_{sLDn} = 0 \quad [5\text{-}121]$$

$$\frac{1}{R_{LDb}} V_{sbg} - I_{sLDb} - \frac{1}{R_{LDb}} V_{sLDn} = 0 \quad [5\text{-}122]$$

$$\frac{1}{R_{LDc}} V_{scg} - I_{sLDc} - \frac{1}{R_{LDc}} V_{sLDn} = 0 \quad [5\text{-}123]$$

Match up the load currents to the transformer secondary currents

$$I_{sa} + I_{sLDa} = 0 \quad [5\text{-}124]$$

$$I_{sb} + I_{sLDb} = 0 \quad [5\text{-}125]$$

$$I_{sc} + I_{sLDc} = 0 \qquad [5\text{-}126]$$

$$I_{sLDa} + I_{sLDb} + I_{sLDc} - I_{sLDn} = 0 \qquad [5\text{-}127]$$

Since this is a 4-wire system, connect the voltages of the neutral conductors of the load and secondary windings

$$V_{sng} - V_{sLDn} = 0 \qquad [5\text{-}128]$$

This set of 28 equations can be solved for the following set of 28 variables:

$$I_{pa} = -\frac{(R_{LDb}V_C+R_{LDc}V_B+(-R_{LDc}-R_{LDb})V_A)n^2+R_T(V_C+V_B-2V_A)}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_Tn^2+3R_T^{\,2}} \qquad [5\text{-}1a]$$

$$I_{pb} = -\frac{(R_{LDa}V_C+(-R_{LDc}-R_{LDa})V_B+R_{LDc}V_A)n^2+R_T(V_C-2V_B+V_A)}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_Tn^2+3R_T^{\,2}} \qquad [5\text{-}2a]$$

$$I_{pc} = -\frac{((-R_{LDb}-R_{LDa})V_C+R_{LDa}V_B+R_{LDb}V_A)n^2+R_T(-2V_C+V_B+V_A)}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_Tn^2+3R_T^{\,2}} \qquad [5\text{-}3a]$$

$$I_{pn} = 0 \qquad [5\text{-}4a]$$

$$V_{tpa} = \frac{(-R_{LDa}R_{LDb}V_C-R_{LDa}R_{LDc}V_B+R_{LDa}(R_{LDc}+R_{LDb})V_A)n^4+}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_Tn^2+3R_T^{\,2}} + \frac{R_T\big((-R_{LDb}-R_{LDa})V_C+(-R_{LDc}-R_{LDa})V_B+(R_{LDc}+R_{LDb}+2R_{LDa})V_A\big)n^2+R_T^{\,2}(-V_C-V_B+2V_A)}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_Tn^2+3R_T^{\,2}} \qquad [5\text{-}5a]$$

$$V_{tpb} = -\frac{(R_{LDa}R_{LDb}V_C+(-R_{LDb}R_{LDc}-R_{LDa}R_{LDb})V_B+R_{LDb}R_{LDc}V_A)n^4}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_Tn^2+3R_T^{\,2}} - \frac{R_T((R_{LDb}+R_{LDa})V_C+(-R_{LDc}-2R_{LDb}-R_{LDa})V_B+(R_{LDc}+R_{LDb})V_A)n^2+R_T^{\,2}(V_C-2V_B+V_A)}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_Tn^2+3R_T^{\,2}} \qquad [5\text{-}6a]$$

$$V_{tpc} = -\frac{((-R_{LDb}R_{LDc}-R_{LDa}R_{LDc})V_C+R_{LDa}R_{LDc}V_B+R_{LDb}R_{LDc}V_A)n^4}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_Tn^2+3R_T^{\,2}} - \frac{R_T((-2R_{LDc}-R_{LDb}-R_{LDa})V_C+(R_{LDc}+R_{LDa})V_B+(R_{LDc}+R_{LDb})V_A)n^2+R_T^{\,2}(-2V_C+V_B+V_A)}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_Tn^2+3R_T^{\,2}} \qquad [5\text{-}7a]$$

$$V_{tsa} = -\frac{(R_{LDa}R_{LDb}V_C+R_{LDa}R_{LDc}V_B+(-R_{LDa}R_{LDc}-R_{LDa}R_{LDb})V_A)n^3+R_T(R_{LDa}V_C+R_{LDa}V_B-2R_{LDa}V_A)n}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_Tn^2+3R_T^{\,2}} \qquad [5\text{-}8a]$$

$$V_{tsb} = -\frac{(R_{LDa}R_{LDb}V_C+(-R_{LDb}R_{LDc}-R_{LDa}R_{LDb})V_B+R_{LDb}R_{LDc}V_A)n^3+R_T(R_{LDb}V_C-2R_{LDb}V_B+R_{LDb}V_A)n}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_Tn^2+3R_T^{\,2}} \qquad [5\text{-}9a]$$

$$V_{tsc} = -\frac{((-R_{LDb}R_{LDc}-R_{LDa}R_{LDc})V_C+R_{LDa}R_{LDc}V_B+R_{LDb}R_{LDc}V_A)n^3+R_T(-2R_{LDc}V_C+R_{LDc}V_B+R_{LDc}V_A)n}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_Tn^2+3R_T^{\,2}} \qquad [5\text{-}10a]$$

$$I_{sa} = -\frac{(-R_{LDb}V_C-R_{LDc}V_B+(R_{LDc}+R_{LDb})V_A)n^3+R_T(-V_C-V_B+2V_A)n}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_Tn^2+3R_T^{\,2}} \qquad [5\text{-}11a]$$

$$I_{sb} = \frac{(R_{LDa}V_C+(-R_{LDc}-R_{LDa})V_B+R_{LDc}V_A)n^3+R_T(V_C-2V_B+V_A)n}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_Tn^2+3R_T^{\,2}} \qquad [5\text{-}12a]$$

$$I_{sc} = \frac{((-R_{LDb}-R_{LDa})V_C+R_{LDa}V_B+R_{LDb}V_A)n^3+R_T(-2V_C+V_B+V_A)n}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_Tn^2+3R_T^{\,2}} \qquad [5\text{-}13a]$$

$$I_{sn} = 0 \qquad [5\text{-}14a]$$

$$V_{pag} = V_A \qquad [5\text{-}15a]$$

$$V_{pbg} = V_B \quad [5\text{-}16a]$$

$$V_{pcg} = V_C \quad [5\text{-}17a]$$

$$V_{png} = \frac{(R_{LDa}R_{LDb}V_C + R_{LDa}R_{LDc}V_B + R_{LDb}R_{LDc}V_A)n^4}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_Tn^2+3R_T{}^2} + \frac{R_T((R_{LDb}+R_{LDa})V_C+(R_{LDc}+R_{LDa})V_B+(R_{LDc}+R_{LDb})V_A)n^2+R_T{}^2(V_C+V_B+V_A)}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_Tn^2+3R_T{}^2} \quad [5\text{-}18a]$$

$$V_{sag} = -\frac{(R_{LDa}R_{LDb}V_C+R_{LDa}R_{LDc}V_B+R_{LDa}(-R_{LDc}-R_{LDb})V_A)n^3+R_T(R_{LDa}V_C+R_{LDa}V_B-2R_{LDa}V_A)n}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_Tn^2+3R_T{}^2} \quad [5\text{-}19a]$$

$$V_{sbg} = -\frac{(R_{LDa}R_{LDb}V_C+R_{LDb}(-R_{LDc}-R_{LDa})V_B+R_{LDb}R_{LDc}V_A)n^3+R_T(R_{LDb}V_C-2R_{LDb}V_B+R_{LDb}V_A)n}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_Tn^2+3R_T{}^2} \quad [5\text{-}20a]$$

$$V_{scg} = -\frac{(R_{LDc}(-R_{LDb}-R_{LDa})V_C+R_{LDa}R_{LDc}V_B+R_{LDb}R_{LDc}V_A)n^3+R_T(-2R_{LDc}V_C+R_{LDc}V_B+R_{LDc}V_A)n}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_Tn^2+3R_T{}^2} \quad [5\text{-}21a]$$

$$V_{sng} = 0 \quad [5\text{-}22a]$$

$$V_{CMp} = 0 \quad [5\text{-}23a]$$

$$I_{sLDa} = \frac{(-R_{LDb}V_C-R_{LDc}V_B+(R_{LDc}+R_{LDb})V_A)n^3+R_T(-V_C-V_B+2V_A)n}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_Tn^2+3R_T{}^2} \quad [5\text{-}24a]$$

$$I_{sLDb} = -\frac{(R_{LDa}V_C+(-R_{LDc}-R_{LDa})V_B+R_{LDc}V_A)n^3+R_T(V_C-2V_B+V_A)n}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_Tn^2+3R_T{}^2} \quad [5\text{-}25a]$$

$$I_{sLDc} = -\frac{((-R_{LDb}-R_{LDa})V_C+R_{LDa}V_B+R_{LDb}V_A)n^3+R_T(-2V_C+V_B+V_A)n}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_Tn^2+3R_T{}^2} \quad [5\text{-}26a]$$

$$V_{sLDn} = 0 \quad [5\text{-}27a]$$

$$I_{sLDn} = 0 \quad [5\text{-}28a]$$

If the sources are balanced then

$$V_A + V_B + V_C = 0 \quad [5\text{-}201]$$

With this assumption, a number of the equations can be simplified.

$$I_{pa} = -\frac{(R_{LDb}V_C+R_{LDc}V_B+(-R_{LDc}-R_{LDb})V_A)n^2+R_T(-3V_A)}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_Tn^2+3R_T{}^2} \quad [5\text{-}1b]$$

$$I_{pb} = -\frac{(R_{LDa}V_C+(-R_{LDc}-R_{LDa})V_B+R_{LDc}V_A)n^2+R_T(-3V_B)}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_Tn^2+3R_T{}^2} \quad [5\text{-}2b]$$

$$I_{pc} = -\frac{((-R_{LDb}-R_{LDa})V_C+R_{LDa}V_B+R_{LDb}V_A)n^2+R_T(-3V_C)}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_Tn^2+3R_T{}^2} \quad [5\text{-}3b]$$

$$I_{sa} = -\frac{(-R_{LDb}V_C-R_{LDc}V_B+(R_{LDc}+R_{LDb})V_A)n^3+R_T(3V_A)n}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_Tn^2+3R_T{}^2} \quad [5\text{-}11b]$$

$$I_{sb} = \frac{(R_{LDa}V_C+(-R_{LDc}-R_{LDa})V_B+R_{LDc}V_A)n^3+R_T(-3V_B)n}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_Tn^2+3R_T{}^2} \quad [5\text{-}12b]$$

$$I_{sc} = \frac{((-R_{LDb}-R_{LDa})V_C+R_{LDa}V_B+R_{LDb}V_A)n^3+R_T(-3V_C)n}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_Tn^2+3R_T{}^2} \quad [5\text{-}13b]$$

$$V_{png} = \frac{(R_{LDa}R_{LDb}V_C + R_{LDa}R_{LDc}V_B + R_{LDb}R_{LDc}V_A)n^4}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_T n^2+3{R_T}^2} + \frac{R_T((R_{LDb}+R_{LDa})V_C+(R_{LDc}+R_{LDa})V_B+(R_{LDc}+R_{LDb})V_A)n^2}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_T n^2+3{R_T}^2} \quad [5\text{-}18b]$$

$$V_{sag} = -\frac{(R_{LDa}R_{LDb}V_C+R_{LDa}R_{LDc}V_B+R_{LDa}(-R_{LDc}-R_{LDb})V_A)n^3+R_T(-3R_{LDa}V_A)n}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_T n^2+3{R_T}^2} \quad [5\text{-}19b]$$

$$V_{sbg} = -\frac{(R_{LDa}R_{LDb}V_C+R_{LDb}(-R_{LDc}-R_{LDa})V_B+R_{LDb}R_{LDc}V_A)n^3+R_T(-3R_{LDb}V_B)n}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_T n^2+3{R_T}^2} \quad [5\text{-}20b]$$

$$V_{scg} = -\frac{(R_{LDc}(-R_{LDb}-R_{LDa})V_C+R_{LDa}R_{LDc}V_B+R_{LDb}R_{LDc}V_A)n^3+R_T(-3R_{LDc}V_C)n}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_T n^2+3{R_T}^2} \quad [5\text{-}21b]$$

$$I_{sLDa} = \frac{(-R_{LDb}V_C-R_{LDc}V_B+(R_{LDc}+R_{LDb})V_A)n^3+R_T(3V_A)n}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_T n^2+3{R_T}^2} \quad [5\text{-}24b]$$

$$I_{sLDb} = -\frac{(R_{LDa}V_C+(-R_{LDc}-R_{LDa})V_B+R_{LDc}V_A)n^3+R_T(-3V_B)n}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_T n^2+3{R_T}^2} \quad [5\text{-}25b]$$

$$I_{sLDc} = -\frac{((-R_{LDb}-R_{LDa})V_C+R_{LDa}V_B+R_{LDb}V_A)n^3+R_T(-3V_C)n}{(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_T n^2+3{R_T}^2} \quad [5\text{-}26b]$$

Some conclusions from these equations include:

(a) Under normal conditions, no current flows in the neutral of either transformer.
(b) The voltage at the neutral of the secondary with respect to ground is equal to zero, even when the loads are not balanced.
(c) An unbalanced load results in the voltage of the neutral of the primary with respect to ground being something other than zero. If all the load resistors have an equal value, then the voltage of the neutral of the primary with respect to ground will be zero.
(d) Neither the primary or secondary grounding resistor will have current flowing through them, even with unbalanced load resistors.
(e) The unbalanced load resistance result in the phase currents in the primary to be out of phase with the voltage, resulting in a lower than unity power factor. The currents may not be 120° out of phase with each other.
(f) The secondary voltages are balanced, but phase shifted with respect to the primary voltages.

The equations can be simplified further by assuming ideal transformers, where $R_T = 0$:

$$I_{pa} = -\frac{(R_{LDb}V_C+R_{LDc}V_B+(-R_{LDc}-R_{LDb})V_A)}{(R_{LDa}R_{LDc}+R_{LDa}R_{LDb}+R_{LDb}R_{LDc})n^2} \quad [5\text{-}1c]$$

$$I_{pb} = -\frac{(R_{LDa}V_C+(-R_{LDc}-R_{LDa})V_B+R_{LDc}V_A)}{(R_{LDa}R_{LDc}+R_{LDa}R_{LDb}+R_{LDb}R_{LDc})n^2} \quad [5\text{-}2c]$$

$$I_{pc} = -\frac{((-R_{LDb}-R_{LDa})V_C+R_{LDa}V_B+R_{LDb}V_A)}{(R_{LDa}R_{LDc}+R_{LDa}R_{LDb}+R_{LDb}R_{LDc})n^2} \quad [5\text{-}3c]$$

$$I_{sa} = \frac{(R_{LDb}V_C+R_{LDc}V_B+(-R_{LDc}-R_{LDb})V_A)}{(R_{LDa}R_{LDc}+R_{LDa}R_{LDb}+R_{LDb}R_{LDc})n} \quad [5\text{-}11c]$$

$$I_{sb} = \frac{(R_{LDa}V_C+(-R_{LDc}-R_{LDa})V_B+R_{LDc}V_A)}{(R_{LDa}R_{LDc}+R_{LDa}R_{LDb}+R_{LDb}R_{LDc})n} \quad [5\text{-}12c]$$

$$I_{sc} = \frac{((-R_{LDb}-R_{LDa})V_C+R_{LDa}V_B+R_{LDb}V_A)}{(R_{LDa}R_{LDc}+R_{LDa}R_{LDb}+R_{LDb}R_{LDc})n} \quad [5\text{-}13c]$$

$$V_{png} = \frac{(R_{LDa}R_{LDb}V_C+R_{LDa}R_{LDc}V_B+R_{LDb}R_{LDc}V_A)}{(R_{LDa}R_{LDc}+R_{LDa}R_{LDb}+R_{LDb}R_{LDc})} \quad [5\text{-}18c]$$

$$V_{sag} = -\frac{(R_{LDa}R_{LDb}V_C + R_{LDa}R_{LDc}V_B + R_{LDa}(-R_{LDc} - R_{LDb})V_A)}{(R_{LDa}R_{LDc} + R_{LDa}R_{LDb} + R_{LDb}R_{LDc})n} \quad [5\text{-}19c]$$

$$V_{sbg} = -\frac{(R_{LDa}R_{LDb}V_C + R_{LDb}(-R_{LDc} - R_{LDa})V_B + R_{LDb}R_{LDc}V_A)}{(R_{LDa}R_{LDc} + R_{LDa}R_{LDb} + R_{LDb}R_{LDc})n} \quad [5\text{-}20c]$$

$$V_{scg} = -\frac{(R_{LDc}(-R_{LDb} - R_{LDa})V_C + R_{LDa}R_{LDc}V_B + R_{LDb}R_{LDc}V_A)}{(R_{LDa}R_{LDc} + R_{LDa}R_{LDb} + R_{LDb}R_{LDc})n} \quad [5\text{-}21c]$$

$$I_{sLDa} = \frac{(-R_{LDb}V_C - R_{LDc}V_B + (R_{LDc} + R_{LDb})V_A)}{(R_{LDa}R_{LDc} + R_{LDa}R_{LDb} + R_{LDb}R_{LDc})n} \quad [5\text{-}24c]$$

$$I_{sLDb} = \frac{(-R_{LDa}V_C + (R_{LDc} + R_{LDa})V_B - R_{LDc}V_A)}{(R_{LDa}R_{LDc} + R_{LDa}R_{LDb} + R_{LDb}R_{LDc})n} \quad [5\text{-}25c]$$

$$I_{sLDc} = \frac{((R_{LDb} + R_{LDa})V_C - R_{LDa}V_B - R_{LDb}V_A)}{(R_{LDa}R_{LDc} + R_{LDa}R_{LDb} + R_{LDb}R_{LDc})n} \quad [5\text{-}26c]$$

Figure 11 depicts Figure 10 with a ground fault on Phase A of the secondary and with the simplification of a balance load resistance. If we also assume the voltage sources are balanced ($V_A + V_B + V_C = 0$), then the following equations are modified:

$$V_{pcg} - V_{CMp} = -V_B - V_A \quad [5\text{-}117d]$$

$$\frac{1}{R_{LD}}V_{sag} - I_{sLDa} - \frac{1}{R_{LD}}V_{sLDn} = 0 \quad [5\text{-}121d]$$

$$\frac{1}{R_{LD}}V_{sbg} - I_{sLDb} - \frac{1}{R_{LD}}V_{sLDn} = 0 \quad [5\text{-}122d]$$

$$\frac{1}{R_{LD}}V_{scg} - I_{sLDc} - \frac{1}{R_{LD}}V_{sLDn} = 0 \quad [5\text{-}123d]$$

$$I_{sa} + I_{sLDa} + I_{sfault} = 0 \quad [5\text{-}124d]$$

The following equation is added:

$$V_{sag} = 0 \quad [5\text{-}129d]$$

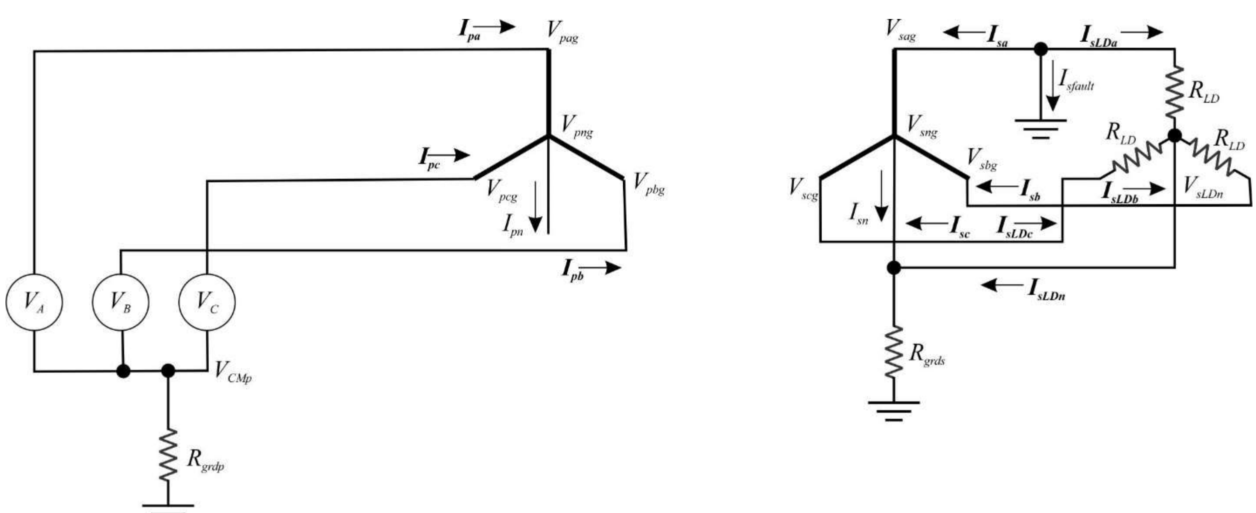


Figure 11: Wye-wye transformer with source and load and ground fault of secondary Phase A.

The resulting modified system of equations can be solved symbolically:

$$I_{pa} = \frac{(3R_{grds}+3R_{LD})V_A}{(3R_{LD}R_{grds}+R_{LD}^2)n^2+R_T(3R_{grds}+3R_{LD})} \tag{5-1e}$$

$$I_{pb} = -\frac{((-3R_{LD}R_{grds}-R_{LD}^2)V_B+R_{LD}^2V_A)n^2+R_T(-3R_{grds}-3R_{LD})V_B}{(3R_{LD}^2R_{grds}+R_{LD}^3)n^4+R_T(6R_{LD}R_{grds}+4R_{LD}^2)n^2+R_T^2(3R_{grds}+3R_{LD})} \tag{5-2e}$$

$$I_{pc} = -\frac{\left((-3R_{LD}R_{grds}-R_{LD}^2)V_C+R_{LD}^2V_A\right)n^2+R_T\left(-3R_{grds}-3R_{LD}\right)V_C}{(3R_{LD}^2R_{grds}+R_{LD}^3)n^4+R_T(6R_{LD}R_{grds}+4R_{LD}^2)n^2+R_T^2(3R_{grds}+3R_{LD})} \tag{5-3e}$$

$$I_{pn} = 0 \tag{5-4e}$$

$$V_{tpa} = \frac{3R_{LD}R_{grds}V_An^2+R_T(3R_{grds}+3R_{LD})V_A}{(3R_{LD}R_{grds}+R_{LD}^2)n^2+R_T(3R_{grds}+3R_{LD})} \tag{5-5e}$$

$$V_{tpb} = -\frac{((-3R_{LD}R_{grds}-R_{LD}^2)V_B+R_{LD}^2V_A)n^2+R_T(-3R_{grds}-3R_{LD})V_B}{(3R_{LD}R_{grds}+R_{LD}^2)n^2+R_T(3R_{grds}+3R_{LD})} \tag{5-6e}$$

$$V_{tpc} = -\frac{((-3R_{LD}R_{grds}-R_{LD}^2)V_C+R_{LD}^2V_A)n^2+R_T(-3R_{grds}-3R_{LD})V_C}{(3R_{LD}R_{grds}+R_{LD}^2)n^2+R_T(3R_{grds}+3R_{LD})} \tag{5-7e}$$

$$V_{tsa} = \frac{3R_{LD}R_{grds}V_An}{(3R_{LD}R_{grds}+R_{LD}^2)n^2+R_T(3R_{grds}+3R_{LD})} \tag{5-8e}$$

$$V_{tsb} = -\frac{((-3R_{LD}^2R_{grds}-R_{LD}^3)V_B+R_{LD}^3V_A)n^3+R_T(-3R_{LD}R_{grds}-3RR_{LD}^2)V_Bn}{(3R_{LD}^2R_{grds}+R_{LD}^3)n^4+R_T(6R_{LD}R_{grds}+4R_{LD}^2)n^2+R_T^2(3R_{grds}+3R_{LD})} \tag{5-9e}$$

$$V_{tsc} = -\frac{((-3R_{LD}^2R_{grds}-R_{LD}^3)V_C+R_{LD}^3V_A)n^3+R_T(-3R_{LD}R_{grds}-3R_{LD}^2)V_Cn}{(3R_{LD}^2R_{grds}+R_{LD}^3)n^4+R_T(6R_{LD}R_{grds}+4R_{LD}^2)n^2+R_T^2(3R_{grds}+3R_{LD})} \tag{5-10e}$$

$$I_{sa} = -\frac{(3R_{grds}+3R_{LD})V_An}{(3R_{LD}R_{grds}+R_{LD}^2)n^2+R_T(3R_{grds}+3R_{LD})} \tag{5-11e}$$

$$I_{sb} = \frac{((-3R_{LD}R_{grds}-R_{LD}^2)V_B+R_{LD}^2V_A)n^3+R_T(-3R_{grds}-3R_{LD})V_Bn}{(3R_{LD}^2R_{grds}+R_{LD}^3)n^4+R_T(6R_{LD}R_{grds}+4R_{LD}^2)n^2+R_T^2(3R_{grds}+3R_{LD})} \tag{5-12e}$$

$$I_{sc} = \frac{((-3R_{LD}R_{grds}-R_{LD}^2)V_C+R_{LD}^2V_A)n^3+R_T(-3R_{grds}-3R_{LD})V_Cn}{(3R_{LD}^2R_{grds}+R_{LD}^3)n^4+R_T(6R_{LD}R_{grds}+4R_{LD}^2)n^2+R_T^2(3R_{grds}+3R_{LD})} \tag{5-13e}$$

$$I_{sn} = 0 \tag{5-14e}$$

$$V_{pag} = V_A \tag{5-15e}$$

$$V_{pbg} = V_B \tag{5-16e}$$

$$V_{pcg} = -V_B - V_A = V_C \tag{5-17e}$$

$$V_{png} = \frac{R_{LD}^2V_An^2}{(3R_{LD}R_{grds}+R_{LD}^2)n^2+R_T(3R_{grds}+3R_{LD})} \tag{5-18e}$$

$$V_{sag} = 0 \tag{5-19e}$$

$$V_{sbg} = -\frac{((-3R_{LD}^2R_{grds}-R_{LD}^3)V_B+(3R_{LD}^2R_{grds}+R_{LD}^3)V_A)n^3+R_T((-3R_{LD}R_{grds}-3R_{LD}^2)V_B+3R_{LD}R_{grds}V_A)n}{(3R_{LD}^2R_{grds}+R_{LD}^3)n^4+R_T(6R_{LD}R_{grds}+4R_{LD}^2)n^2+R_T^2(3R_{grds}+3R_{LD})} \tag{5-20e}$$

$$V_{scg} = -\frac{((-3R_{LD}^2R_{grds}-R_{LD}^3)V_C+(3R_{LD}^2R_{grds}+R_{LD}^3)V_A)n^3+R_T((-3R_{LD}R_{grds}-3R_{LD}^2)V_C+3R_{LD}R_{grds}V_A)n}{(3R_{LD}^2R_{grds}+R_{LD}^3)n^4+R_T(6R_{LD}R_{grds}+4R_{LD}^2)n^2+R_T^2(3R_{grds}+3R_{LD})} \tag{5-21e}$$

$$V_{sng} = -\frac{3R_{LD}R_{grds}V_An}{(3R_{LD}R_{grds}+R_{LD}^2)n^2+R_T(3R_{grds}+3R_{LD})} \tag{5-22e}$$

$$V_{CMp} = 0 \qquad [5\text{-}23e]$$

$$I_{sLDa} = \frac{3R_{grds}V_A n}{(3R_{LD}R_{grds}+R_{LD}{}^2)n^2+R_T(3R_{grds}+3R_{LD})} \qquad [5\text{-}24e]$$

$$I_{sLDb} = -\frac{((-3R_{LD}R_{grds}-R_{LD}{}^2)V_B+R_{LD}{}^2V_A)n^3+R_T(-3R_{grds}-3R_{LD})V_B n}{(3R_{LD}{}^2R_{grds}+R_{LD}{}^3)n^4+R_T(6R_{LD}R_{grds}+4R_{LD}{}^2)n^2+R_T{}^2(3R_{grds}+3R_{LD})} \qquad [5\text{-}25e]$$

$$I_{sLDc} = -\frac{((-3R_{LD}R_{grds}-R_{LD}{}^2)V_C+R_{LD}{}^2V_A)n^3+R_T(-3R_{grds}-3R_{LD})V_C n}{(3R_{LD}{}^2R_{grds}+R_{LD}{}^3)n^4+R_T(6R_{LD}R_{grds}+4R_{LD}{}^2)n^2+R_T{}^2(3R_{grds}+3R_{LD})} \qquad [5\text{-}26e]$$

$$V_{sLDn} = -\frac{3R_{LD}R_{grds}V_A n}{(3R_{LD}R_{grds}+R_{LD}{}^2)n^2+R_T(3R_{grds}+3R_{LD})} \qquad [5\text{-}27e]$$

$$I_{sLDn} = -\frac{3R_{LD}V_A n}{(3R_{LD}R_{grds}+R_{LD}{}^2)n^2+R_T(3R_{grds}+3R_{LD})} \qquad [5\text{-}28e]$$

$$I_{sfault} = \frac{3R_{LD}V_A n}{(3R_{LD}R_{grds}+R_{LD}{}^2)n^2+R_T(3R_{grds}+3R_{LD})} \qquad [5\text{-}29e]$$

For a four-wire system using wye-wye transformers that experiences a ground fault, the following observations can be made:

(a) Under a ground fault conditions, no current flows in the neutral of either transformer.
(b) The neutral conductor on the secondary will have a voltage proportional to the faulted phase voltage.
(c) The primary and secondary currents for the faulted phase will be in phase with the phase voltage.
(d) The primary and secondary currents for the unfaulted phases will not be in phase with their respective phase voltages.
(e) The secondary grounding resistor impacts the primary and secondary voltages and currents.
(f) If the load resistance is significantly higher than the grounding resistor resistance (lightly loaded), then the fault current will be limited by the load resistance
(g) If the load resistance is significantly less than the grounding resistor resistance, then the grounding resistor will limit the fault current.

If the secondary winding is solidly grounded, then $R_{grds} = 0$. if $R_{LD} \gg R_T$The solutions can be simplified to:

$$I_{pa} = \frac{3V_A}{R_{LD}n^2} \qquad [5\text{-}1f]$$

$$I_{pb} = \frac{(V_B-V_A)}{R_{LD}n^2} \qquad [5\text{-}2f]$$

$$I_{pc} = \frac{(V_C-V_A)}{R_{LD}n^2} \qquad [5\text{-}3f]$$

$$I_{sa} = -\frac{3V_A}{R_{LD}n} \qquad [5\text{-}11f]$$

$$I_{sb} = \frac{(V_A-V_B)}{R_{LD}n} \qquad [5\text{-}12f]$$

$$I_{sc} = \frac{(V_A-V_C)}{R_{LD}n} \qquad [5\text{-}13f]$$

$$V_{sag} = 0 \qquad [5\text{-}19f]$$

$$V_{sbg} = \frac{(V_B-V_A)}{n} \qquad [5\text{-}20f]$$

$$V_{scg} = \frac{(V_C-V_A)}{n} \qquad [5\text{-}21f]$$

$$V_{sng} = 0 \qquad [5\text{-}22f]$$

$$I_{sLDa} = 0 \quad [5\text{-}24f]$$

$$I_{sLDb} = \frac{(V_B - V_A)}{R_{LD}n} \quad [5\text{-}25f]$$

$$I_{sLDc} = \frac{(V_C - V_A)}{R_{LD}n} \quad [5\text{-}26f]$$

$$V_{sLDn} = 0 \quad [5\text{-}27f]$$

$$I_{sLDn} = -\frac{3V_A}{R_{LD}n} \quad [5\text{-}28f]$$

$$I_{sfault} = \frac{3V_A}{R_{LD}n} \quad [5\text{-}29f]$$

The fault current is limited to roughly 3 times the unfaulted secondary line current.

On the other hand, if the secondary is ungrounded, then $R_{grds} = \infty$ and if $R_{LD} \gg R_T$ the solutions can be simplified to:

$$I_{pa} = \frac{V_A}{R_{LD}n^2} \quad [5\text{-}1g]$$

$$I_{pb} = \frac{V_B}{R_{LD}n^2} \quad [5\text{-}2g]$$

$$I_{pc} = \frac{V_C}{R_{LD}n^2} \quad [5\text{-}3g]$$

$$I_{sa} = -\frac{V_A}{R_{LD}n} \quad [5\text{-}11g]$$

$$I_{sb} = -\frac{V_B}{R_{LD}n} \quad [5\text{-}12g]$$

$$I_{sc} = -\frac{V_C}{R_{LD}n} \quad [5\text{-}13g]$$

$$V_{sag} = 0 \quad [5\text{-}19e]$$

$$V_{sbg} = \frac{(V_B - V_A)}{n} \quad [5\text{-}20g]$$

$$V_{scg} = \frac{(V_C - V_A)}{n} \quad [5\text{-}21g]$$

$$V_{sng} = -\frac{V_A}{n} \quad [5\text{-}22g]$$

$$V_{CMp} = 0 \quad [5\text{-}23g]$$

$$I_{sLDa} = \frac{V_A}{R_{LD}n} \quad [5\text{-}24g]$$

$$I_{sLDb} = \frac{V_B}{R_{LD}n} \quad [5\text{-}25g]$$

$$I_{sLDc} = \frac{V_C}{R_{LD}n} \quad [5\text{-}26g]$$

$$V_{sLDn} = -\frac{V_A}{n} \quad [5\text{-}27g]$$

$$I_{sLDn} = 0 \quad [5\text{-}28g]$$

$$I_{sfault} = 0 \quad [5\text{-}29g]$$

As expected, the only indication of a ground fault is that all the line to ground voltages are shifted by the voltage of the faulted conductor.

Figure 12 depicts a wye-wye transformer connected to a 3-wire delta connected load and a ground fault on phase A. The neutral conductor of the secondary is grounded through a grounding resistor

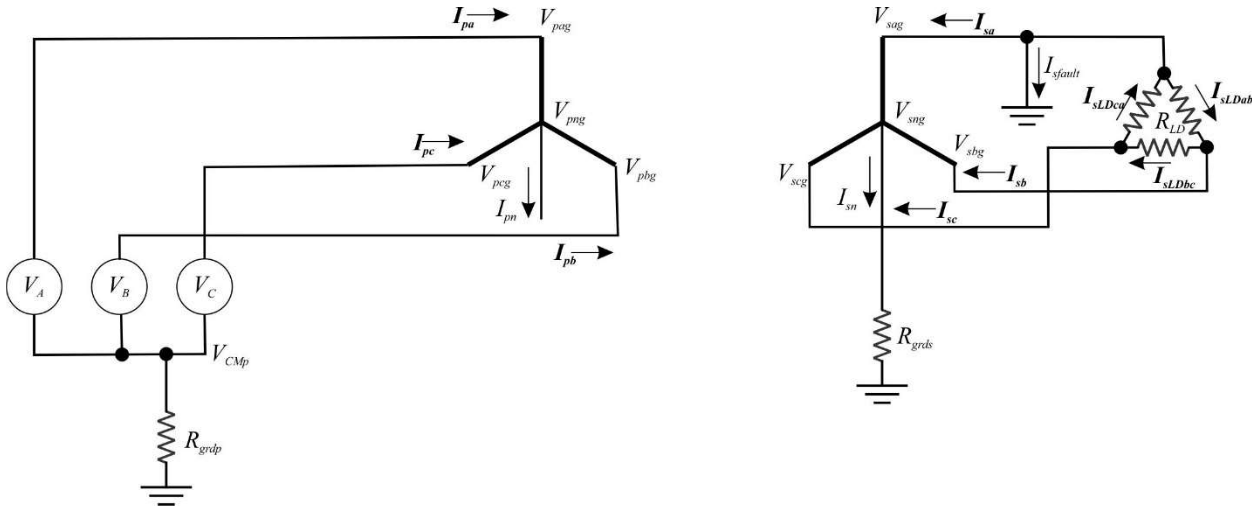


Figure 12: Wye-wye transformer with 3 wire delta connected load

The following equations are modified:

Assume a 3 wire system on the secondary with a grounding resistor.

$$R_{grds} I_{sn} - V_{sng} = 0 \qquad [5\text{-}120h]$$

Assume a delta resistive load where each of the phase resistances is identical

$$V_{sag} - V_{sbg} - R_{LD} I_{sLDab} = 0 \qquad [5\text{-}121h]$$

$$V_{sbg} - V_{scg} - R_{LD} I_{sLDbc} = 0 \qquad [5\text{-}122h]$$

$$V_{scg} - V_{sag} - R_{LD} I_{sLDca} = 0 \qquad [5\text{-}123h]$$

Match up the load currents to the transformer secondary currents. Note that the load currents are different from the wye-load configurations from before.

$$I_{sa} + I_{sLDab} - I_{sLDca} + I_{sfault} = 0 \qquad [5\text{-}124h]$$

$$I_{sb} + I_{sLDbc} - I_{sLDab} = 0 \qquad [5\text{-}125h]$$

$$I_{sc} + I_{sLDca} - I_{sLDbc} = 0 \qquad [5\text{-}126h]$$

Delete the equations for the neutral current and voltage of the load (equations 27 and 28)

And insert the fault equation …

$$V_{sag} = 0 \qquad [5\text{-}129h]$$

The system variables are solved:

$$I_{pa} = \frac{3V_A}{R_{LD}n^2+3R_T} \quad [5\text{-}1i]$$

$$I_{pb} = \frac{3V_B}{R_{LD}n^2+3R_T} \quad [5\text{-}2i]$$

$$I_{pc} = \frac{3V_C}{R_{LD}n^2+3R_T} \quad [5\text{-}3i]$$

$$I_{pn} = 0 \quad [5\text{-}4i]$$

$$I_{sa} = -\frac{3V_A n}{R_{LD}n^2+3R_T} \quad [5\text{-}11i]$$

$$I_{sb} = -\frac{3V_B n}{R_{LD}n^2+3R_T} \quad [5\text{-}12i]$$

$$I_{sc} = -\frac{3V_C n}{R_{LD}n^2+3R_T} \quad [5\text{-}13i]$$

$$V_{png} = \frac{R_{LD}V_A n^2}{R_{LD}n^2+3R_T} \quad [5\text{-}18i]$$

$$V_{sag} = 0 \quad [5\text{-}19i]$$

$$V_{sbg} = -\frac{(R_{LD}V_A - R_{LD}V_B)n}{R_{LD}n^2+3R_T} \quad [5\text{-}20i]$$

$$V_{scg} = -\frac{(R_{LD}V_A - R_{LD}V_C)n}{R_{LD}n^2+3R_T} \quad [5\text{-}21i]$$

$$V_{sng} = 0 \quad [5\text{-}22i]$$

$$V_{CMp} = 0 \quad [5\text{-}23i]$$

$$I_{sLDab} = \frac{(V_A - V_B)n}{R_{LD}n^2+3R_T} \quad [5\text{-}24i]$$

$$I_{sLDbc} = \frac{(V_B - V_C)n}{R_{LD}n^2+3R_T} \quad [5\text{-}25i]$$

$$I_{sLDca} = \frac{(V_C - V_A)n}{R_{LD}n^2+3R_T} \quad [5\text{-}26i]$$

$$I_{sfault} = 0 \quad [5\text{-}29i]$$

For a three wire system using wye-wye transformers that experiences a ground fault, the following observations can be made:

(a) The grounding resistor on the secondary has no impact on ground fault current.
(b) The system behaves as if the system were ungrounded; The primary line currents, secondary line currents, and load currents are not impacted by the ground fault.
(c) The voltage of the neutral conductor of the primary is almost equal to the phase A primary voltage, resulting in very little voltage across the Phase A transformer.
(d) Under a ground fault, the wye-wye transformer essentially behaves as VV (otherwise known as a broken delta transformer). The unfaulted phase transformers are at risk of being overloaded due to the faulted phase (in this case Phase A) transformer not contributing any power.

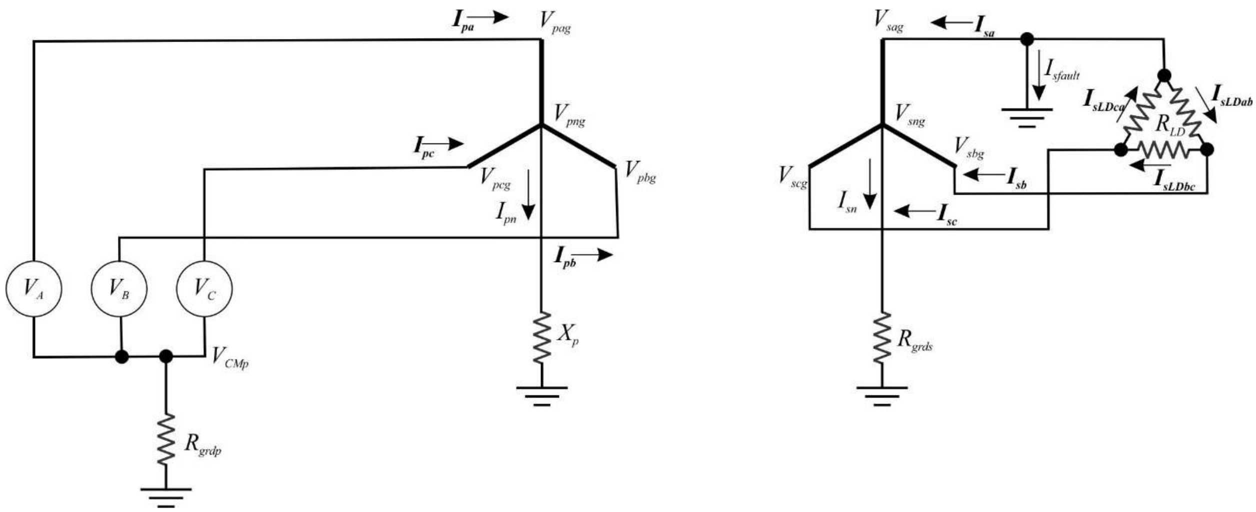


Figure 13: Wye-wye transformer with 3 wire delta connected load and primary neutral grounded

If we assume the primary neutral conductor is connected to ground via an impedance (which is likely to be a complex number) X_p as depicted in Figure 13, then the previous set of equations is modified:

$$X_p I_{pn} - V_{png} = 0 \qquad [5\text{-}119j]$$

X_p may be used to represent the winding capacitance to ground.

The solution to this set of equations is given by:

$$I_{pa} = \frac{(3R_{grds}+R_{LD})V_A n^2+V_A(3X_p+3R_{grdp})+R_T V_A}{R_{LD}R_{grds}n^4+(R_{LD}X_p+R_T(3R_{grds}+R_{LD})+R_{LD}R_{grdp})n^2+R_T(3X_p+3R_{grdp})+{R_T}^2} \qquad [5\text{-}1k]$$

$$I_{pb} = \frac{3R_{LD}R_{grds}V_B n^4+(V_B(3R_{LD}X_p+3R_{LD}R_{grdp})+R_T((9R_{grds}+3R_{LD})V_B+R_{LD}V_A))n^2+R_T V_B(9X_p+9R_{grdp})+3{R_T}^2 V_B}{{R_{LD}}^2R_{grds}n^6+({R_{LD}}^2X_p+R_T(6R_{LD}R_{grds}+{R_{LD}}^2)+{R_{LD}}^2R_{grdp})n^4+(R_T(6R_{LD}X_p+6R_{LD}R_{grdp})+{R_T}^2(9R_{grdp}+4R_{LD}))n^2+{R_T}^2(9X_p+9R_{grdp})+3{R_T}^3} \qquad [5\text{-}2k]$$

$$I_{pc} = \frac{3R_{LD}R_{grds}V_C n^4+(V_C(3R_{LD}X_p+3R_{LD}R_{grdp})+R_T((9R_{grds}+3R_{LD})V_C+R_{LD}V_A))n^2+R_T V_C(9X_p+9R_{grdp})+3{R_T}^2 V_C}{{R_{LD}}^2R_{grds}n^6+({R_{LD}}^2X_p+R_T(6R_{LD}R_{grds}+{R_{LD}}^2)+{R_{LD}}^2R_{grdp})n^4+(R_T(6R_{LD}X_p+6R_{LD}R_{grdp})+{R_T}^2(9R_{grdp}+4R_{LD}))n^2+{R_T}^2(9X_p+9R_{grdp})+3{R_T}^3} \qquad [5\text{-}3k]$$

$$I_{pn} = \frac{R_{LD}V_A n^2}{R_{LD}R_{grds}n^4+(R_{LD}X_p+R_T(3R_{grds}+R_{LD})+R_{LD}R_{grdp})n^2+R_T(3X_p+3R_{grdp})+{R_T}^2} \qquad [5\text{-}4k]$$

$$I_{sa} = -\frac{(3R_{grds}+R_{LD})V_A n^3+(V_A(3X_p+3R_{grds})+R_T V_A)n}{R_{LD}R_{grds}n^4+(R_{LD}X_p+R_T(3R_{grds}+R_{LD})+R_{LD}R_{grdp})n^2+R_T(3X_p+3R_{grdp})+{R_T}^2} \qquad [5\text{-}11k]$$

$$I_{sb} = -\frac{3R_{LD}R_{grds}V_B n^5+(V_B(3R_{LD}Xp+3R_{LD}R_{grdp})+R_T((9R_{grds}+3R_{LD})V_B+R_{LD}V_A))n^3+(R_T V_B(9Xp+9\ \ {}_{grdp})+3{R_T}^2V_B)n}{{R_{LD}}^2R_{grds}n^6+({R_{LD}}^2X_p+R_T(6R_{LD}R_{grds}+{R_{LD}}^2)+{R_{LD}}^2R_{grdp})n^4+(R_T(6R_{LD}X_p+6R_{LD}R_{grdp})+{R_T}^2(9R_{grdp}+4R_{LD}))n^2+{R_T}^2(9X_p+9R_{grdp})+3{R_T}^3} \qquad [5\text{-}12k]$$

$$I_{sc} = -\frac{3R_{LD}R_{grds}V_C n^5+(V_C(3R_{LD}Xp+3R_{LD}R_{grdp})+R_T((9R_{grds}+3R_{LD})V_C+R_{LD}V_A))n^3+(R_T V_C(9Xp+9\ \ {}_{grdp})+3{R_T}^2V_C)n}{{R_{LD}}^2R_{grds}n^6+({R_{LD}}^2X_p+R_T(6R_{LD}R_{grds}+{R_{LD}}^2)+{R_{LD}}^2R_{grdp})n^4+(R_T(6R_{LD}X_p+6R_{LD}R_{grdp})+{R_T}^2(9R_{grdp}+4R_{LD}))n^2+{R_T}^2(9X_p+9R_{grdp})+3{R_T}^3} \qquad [5\text{-}13k]$$

$$I_{sn} = -\frac{R_{LD}V_A n^3}{R_{LD}R_{grds}n^4+(R_{LD}X_p+R_T(3R_{grds}+R_{LD})+R_{LD}R_{grdp})n^2+R_T(3X_p+3R_{grdp})+{R_T}^2} \qquad [5\text{-}14k]$$

$$V_{pag} = \frac{R_{LD}R_{grds}V_A n^4+(R_{LD}V_A X_p+R_T(3R_{grds}+R_{LD})V_A)n^2+R_T V_A(3X_p+3R_{grdp})+{R_T}^2V_A}{R_{LD}R_{grds}n^4+(R_{LD}X_p+R_T(3R_{grds}+R_{LD})+R_{LD}R_{grdp})n^2+R_T(3X_p+3R_{grdp})+{R_T}^2} \qquad [5\text{-}15k]$$

$$V_{pbg} = \frac{R_{LD}R_{grds}V_B n^4+(V_B(R_{LD}X_p+R_{LD}R_{grdp})+R_T(3R_{grds}+R_{LD})V_B-R_{LD}R_{grdp}V_A)n^2+R_T V_B(3X_p+3R_{grdp})+{R_T}^2V_B}{R_{LD}R_{grds}n^4+(R_{LD}X_p+R_T(3R_{grds}+R_{LD})+R_{LD}R_{grdp})n^2+R_T(3X_p+3R_{grdp})+{R_T}^2} \qquad [5\text{-}16k]$$

$$V_{pcg} = \frac{R_{LD}R_{grds}V_C n^4+(V_C(R_{LD}X_p+R_{LD}R_{grdp})+R_T(3R_{grds}+R_{LD})V_C-R_{LD}R_{grdp}V_A)n^2+R_T V_C(3X_p+3R_{grdp})+{R_T}^2V_C}{R_{LD}R_{grds}n^4+(R_{LD}X_p+R_T(3R_{grds}+R_{LD})+R_{LD}R_{grdp})n^2+R_T(3X_p+3R_{grdp})+{R_T}^2} \qquad [5\text{-}17k]$$

$$V_{png} = \frac{R_{LD}V_A X_p n^2}{R_{LD}R_{grds}n^4+(R_{LD}X_p+R_T(3R_{grds}+R_{LD})+R_{LD}R_{grdp})n^2+R_T(3X_p+3R_{grdp})+R_T{}^2} \quad [5\text{-}18k]$$

$$V_{sag} = 0 \quad [5\text{-}19k]$$

$$V_{sbg} = -\frac{(R_{LD}{}^2R_{grds}V_A-R_{LD}{}^2R_{grds}V_B)n^5+\left(V_A(R_{LD}{}^2X_p+R_{LD}{}^2R_{grdp})+V_B(-R_{LD}{}^2X_p-R_{LD}{}^2R_{grdp})+R_T\left((-3R_{LD}R_{grds}-R_{LD}{}^2)V_B+3R_{LD}R_{grds}V_A\right)\right)n^3}{R_{LD}{}^2R_{grds}n^6+(R_{LD}{}^2X_p+R_T(6R_{LD}R_{grds}+R_{LD}{}^2)+R_{LD}{}^2R_{grdp})n^4+\left(R_T(6R_{LD}X_p+6R_{LD}R_{grdp})+R_T{}^2(9R_{grdp}+4R_{LD})\right)n^2+R_T{}^2(9X_p+9R_{grdp})+3R_T{}^3} - \frac{\left(R_T\left(V_A(3R_{LD}X_p+3R_{LD}R_{grdp})+V_B(-3R_{LD}Xp-3R_{LD}R_{grdp})\right)+R_T{}^2(R_{LD}V_A-R_{LD}V_B)\right)n}{R_{LD}{}^2R_{grds}n^6+(R_{LD}{}^2X_p+R_T(6R_{LD}R_{grds}+R_{LD}{}^2)+R_{LD}{}^2R_{grdp})n^4+\left(R_T(6R_{LD}X_p+6R_{LD}R_{grdp})+R_T{}^2(9R_{grdp}+4R_{LD})\right)n^2+R_T{}^2(9X_p+9R_{grdp})+3R_T{}^3} \quad [5\text{-}20k]$$

$$V_{scg} = -\frac{(R_{LD}{}^2R_{grds}V_A-R_{LD}{}^2R_{grds}V_C)n^5+\left(V_A(R_{LD}{}^2X_p+R_{LD}{}^2R_{grdp})+V_C(-R_{LD}{}^2X_p-R_{LD}{}^2R_{grdp})+R_T\left((-3R_{LD}R_{grds}-R_{LD}{}^2)V_C+3R_{LD}R_{grds}V_A\right)\right)n^3}{R_{LD}{}^2R_{grds}n^6+(R_{LD}{}^2X_p+R_T(6R_{LD}R_{grds}+R_{LD}{}^2)+R_{LD}{}^2R_{grdp})n^4+\left(R_T(6R_{LD}X_p+6R_{LD}R_{grdp})+R_T{}^2(9R_{grdp}+4R_{LD})\right)n^2+R_T{}^2(9X_p+9R_{grdp})+3R_T{}^3} - \frac{\left(R_T\left(V_A(3R_{LD}X_p+3R_{LD}R_{grdp})+V_C(-3R_{LD}Xp-3R_{LD}R_{grdp})\right)+R_T{}^2(R_{LD}V_A-R_{LD}V_C)\right)n}{R_{LD}{}^2R_{grds}n^6+(R_{LD}{}^2X_p+R_T(6R_{LD}R_{grds}+R_{LD}{}^2)+R_{LD}{}^2R_{grdp})n^4+\left(R_T(6R_{LD}X_p+6R_{LD}R_{grdp})+R_T{}^2(9R_{grdp}+4R_{LD})\right)n^2+R_T{}^2(9X_p+9R_{grdp})+3R_T{}^3} \quad [5\text{-}21k]$$

$$V_{sng} = -\frac{R_{LD}R_{grds}V_A n^3}{R_{LD}R_{grds}n^4+(R_{LD}X_p+R_T(3R_{grds}+R_{LD})+R_{LD}R_{grdp})n^2+R_T(3X_p+3R_{grdp})+R_T{}^2} \quad [5\text{-}22k]$$

$$V_{CMp} = -\frac{R_{LD}R_{grdp}V_A n^2}{R_{LD}R_{grds}n^4+(R_{LD}X_p+R_T(3R_{grds}+R_{LD})+R_{LD}R_{grdp})n^2+R_T(3X_p+3R_{grdp})+R_T{}^2} \quad [5\text{-}23k]$$

$$I_{sLDab} = \frac{(R_{LD}R_{grds}V_A-R_{LD}R_{grds}V_B)n^5+(V_A(R_{LD}X_p+R_{LD}R_{grdp})+V_B(-R_{LD}X_p-R_{LD}R_{grdp})+R_T((-3R_{grds}-R_{LD})V_B+3R_{grds}V_A))n^3+(R_T(V_A(3X_p+3R_{grdp})+V_B(-3X_p-3R_{grdp}))+R_T{}^2(V_A-V_B))n}{R_{LD}{}^2R_{grds}n^6+(R_{LD}{}^2X_p+R_T(6R_{LD}R_{grds}+R_{LD}{}^2)+R_{LD}{}^2R_{grdp})n^4+(R_T(6R_{LD}X_p+6R_{LD}R_{grdp})+R_T{}^2(9R_{grdp}+4R_{LD}))n^2+R_T{}^2(9X_p+9R_{grdp})+3R_T{}^3} \quad [5\text{-}24k]$$

$$I_{sLDbc} = \frac{(V_B-V_C)n}{R_{LD}n^2+3R_T} \quad [5\text{-}25k]$$

$$I_{sLDca} = \frac{(R_{LD}R_{grds}V_C-R_{LD}R_{grds}V_A)n^5+(V_C(R_{LD}X_p+R_{LD}R_{grdp})+V_A(-R_{LD}X_p-R_{LD}R_{grdp})+R_T((3R_{grds}+R_{LD})V_C-3R_{grds}V_A))n^3+(R_T(V_C(3X_p+3R_{grdp})+V_A(-3X_p-3R_{grdp}))+R_T{}^2(V_C-V_A))n}{R_{LD}{}^2R_{grds}n^6+(R_{LD}{}^2X_p+R_T(6R_{LD}R_{grds}+R_{LD}{}^2)+R_{LD}{}^2R_{grdp})n^4+(R_T(6R_{LD}X_p+6R_{LD}R_{grdp})+R_T{}^2(9R_{grdp}+4R_{LD}))n^2+R_T{}^2(9X_p+9R_{grdp})+3R_T{}^3} \quad [5\text{-}26k]$$

$$I_{sfault} = \frac{R_{LD}V_A n^3}{R_{LD}R_{grds}n^4+(R_{LD}X_p+R_T(3R_{grds}+R_{LD})+R_{LD}R_{grdp})n^2+R_T(3X_p+3R_{grdp})+R_T{}^2} \quad [5\text{-}29k]$$

The solution can be simplified by assuming R_T is negligible which results in the following:

$$I_{pa} = \frac{(3R_{grds}+R_{LD})V_A n^2+V_A(3X_p+3R_{grdp})}{R_{LD}R_{grds}n^4+(R_{LD}X_p+R_{LD}R_{grdp})n^2} \quad [5\text{-}1m]$$

$$I_{pb} = \frac{3R_{grds}V_B n^2+V_B(3X_p+3R_{grdp})}{R_{LD}R_{grds}n^4+(R_{LD}X_p+R_{LD}R_{grdp})n^2} \quad [5\text{-}2m]$$

$$I_{pc} = \frac{3R_{grds}V_C n^2+V_C(3X_p+3R_{grdp})}{R_{LD}R_{grds}n^4+(R_{LD}X_p+R_{LD}R_{grdp})n^2} \quad [5\text{-}3m]$$

$$I_{pn} = \frac{V_A}{R_{grds}n^2+(X_p+R_{grdp})} \quad [5\text{-}4m]$$

$$I_{sa} = -\frac{(3R_{grds}+R_{LD})V_A n^2+V_A(3X_p+3R_{grds})}{R_{LD}R_{grds}n^3+(R_{LD}X_p+R_{LD}R_{grdp})n} \quad [5\text{-}11m]$$

$$I_{sb} = -\frac{3R_{grds}V_B n^2+V_B(3Xp+3\ \ {}_{grdp})}{R_{LD}R_{grds}n^3+(R_{LD}X_p+R_{LD}R_{grdp})n} \quad [5\text{-}12m]$$

$$I_{sc} = -\frac{3R_{grds}V_C n^2 + V_C(3Xp + 3\ \ _{grdp})}{R_{LD}R_{grds}n^3 + (R_{LD}X_p + R_{LD}R_{grdp})n} \quad [5\text{-}13m]$$

$$I_{sn} = -\frac{V_A n}{R_{grds}n^2 + (X_p + R_{grdp})} \quad [5\text{-}14m]$$

$$V_{pag} = \frac{R_{grds}V_A n^2 + V_A X_p}{R_{grds}n^2 + (X_p + R_{grdp})} \quad [5\text{-}15m]$$

$$V_{pbg} = \frac{R_{grds}V_B n^2 + V_B(X_p + R_{grdp}) - R_{grdp}V_A}{R_{grds}n^2 + (X_p + R_{grdp})} \quad [5\text{-}16m]$$

$$V_{pcg} = \frac{R_{grds}V_C n^2 + V_C(X_p + R_{grdp}) - R_{grdp}V_A}{R_{grds}n^2 + (X_p + R_{grdp})} \quad [5\text{-}17m]$$

$$V_{png} = \frac{V_A X_p}{R_{grds}n^2 + (X_p + R_{grdp})} \quad [5\text{-}18m]$$

$$V_{sag} = 0 \quad [5\text{-}19m]$$

$$V_{sbg} = -\frac{(R_{grds}V_A - R_{grds}V_B)n^2 + \left(V_A(X_p + R_{grdp}) + V_B(-X_p - R_{grdp})\right)}{R_{grds}n^3 + (X_p + R_{grdp})n} \quad [5\text{-}20m]$$

$$V_{scg} = -\frac{(R_{grds}V_A - R_{grds}V_C)n^2 + \left(V_A(X_p + R_{grdp}) + V_C(-X_p - R_{grdp})\right)}{R_{grds}n^3 + (X_p + R_{grdp})n} \quad [5\text{-}21m]$$

$$V_{sng} = -\frac{R_{grds}V_A n}{R_{grds}n^2 + (X_p + R_{grdp})} \quad [5\text{-}22m]$$

$$V_{CMp} = -\frac{R_{grdp}V_A}{R_{grds}n^2 + (X_p + R_{grdp})} \quad [5\text{-}23m]$$

$$I_{sLDab} = \frac{(R_{grds}V_A - R_{grds}V_B)n^2 + V_A(X_p + R_{grdp}) + V_B(-X_p - R_{grdp})}{R_{LD}R_{grds}n^3 + (R_{LD}X_p + R_{LD}R_{grdp})n} \quad [5\text{-}24m]$$

$$I_{sLDbc} = \frac{(V_B - V_C)}{R_{LD}n} \quad [5\text{-}25m]$$

$$I_{sLDca} = \frac{(R_{grds}V_C - R_{grds}V_A)n^2 + (V_C(X_p + R_{grdp}) + V_A(-X_p - R_{grdp}))}{R_{LD}R_{grds}n^3 + (R_{LD}X_p + R_{LD}R_{grdp})n} \quad [5\text{-}26m]$$

$$I_{sfault} = \frac{V_A n}{R_{grds}n^2 + (X_p + R_{grdp})} \quad [5\text{-}29m]$$

For a three-wire system using wye-wye transformers that experiences a ground fault and the primary neutral conductor is connected to ground via an impedance, the following observations can be made:

(a) The ground fault current on the secondary depends on all of the grounding resistances and impedances; the common mode currents on the primary and secondary circuits are linked
(b) A ground fault on the secondary will result in a common mode current on the primary side of the transformer.

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6. Wye-Delta Transformers

Three phase wye delta transformers are typically not used in shipboard power systems. This document explores the steady-state performance of three "nearly ideal" transformers when connected into a wye-delta transformer bank. Each transformer is assumed to be an ideal transformer with each transformer winding having a series resistance. If this resistance is assumed to have a complex value, it can represent a general impedance. The wye-delta transformer configurations are provided with ideal voltage sources and with resistive loads. The resistive load value may also be complex. A resistance to ground from the wye neutral conductor of the primary is modeled, but its value can be set to a large number in normal operation where the neutral conductor is not connected to ground. Cases with unbalanced load resistances and with ground faults are analyzed.

Figure 14 depicts the schematic for a wye-delta connected transformer as well as the typical schematic symbol used in many power system drawings. In the schematic symbol, thick parallel lines represent the primary and secondary windings of an equivalent (or actual) single phase transformer. As shown in Chapter 2, the resistances in the primary (R_p) and secondary (R_s) can be combined into a total resistance (R_T) where n is the turns ratio of the single-phase transformers.

$$R_T = R_p + R_s\, n^2 \qquad [2\text{-}107]$$

The turns ratio n is for each of the individual transformers. The relationship of the line-to-line voltages (for balanced three phase voltages) of the primary (V_{pll}) and secondary (V_{sll}) circuits is given by:

$$V_{pln} = V_{pag} - V_{png} = n\left(V_{sag} - V_{sbg}\right) = nV_{sll} \qquad [6\text{-}201]$$

$$V_{pll} = \sqrt{3}V_{pln} = n\sqrt{3}V_{sll} \qquad [6\text{-}202]$$

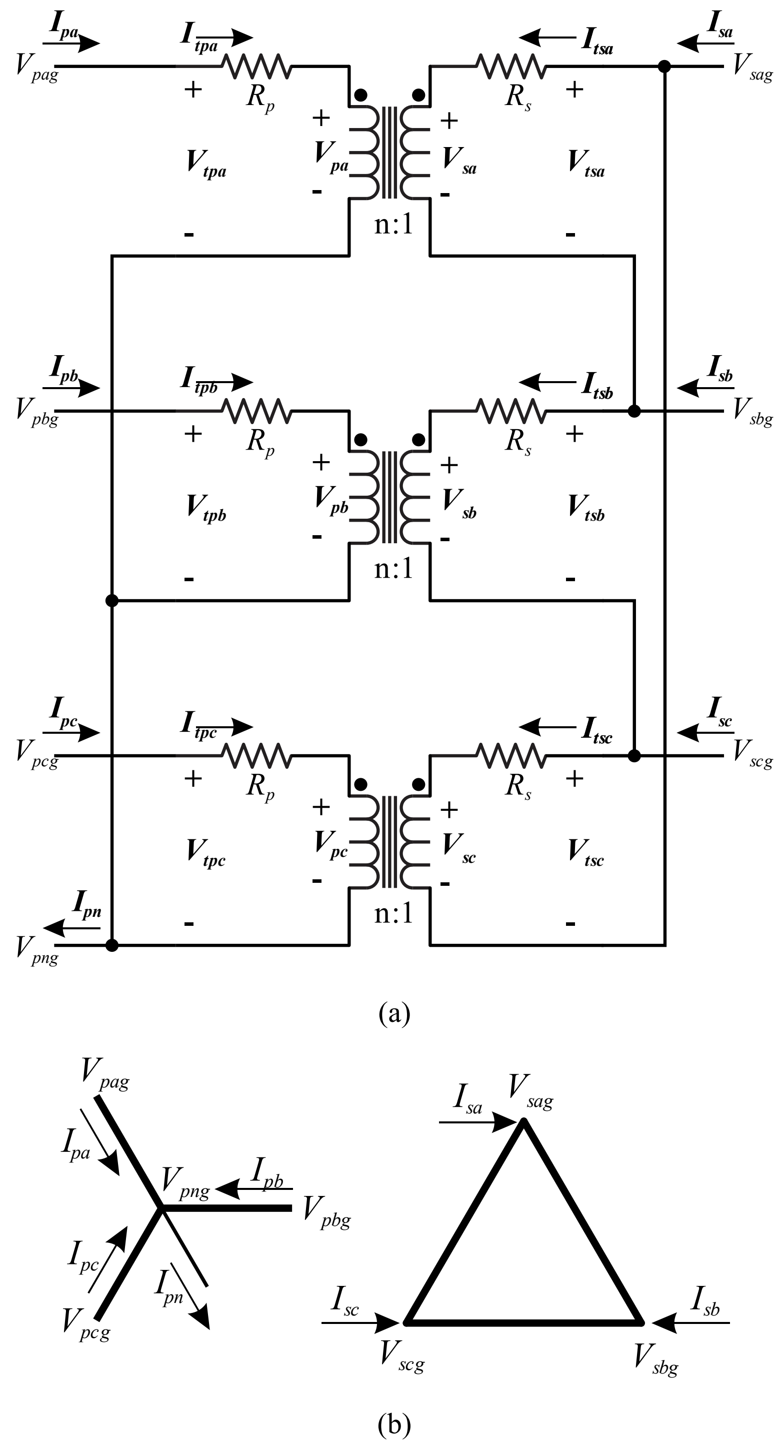

Figure 14: Wye-delta transformer – (a) circuit diagram (b) schematic.

The voltages and currents of the primary and secondary windings are related.

$$I_{pa} - I_{tpa} = 0 \quad [6\text{-}101]$$

$$I_{pb} - I_{tpb} = 0 \quad [6\text{-}102]$$

$$I_{pc} - I_{tpc} = 0 \quad [6\text{-}103]$$

$$nI_{tpa} + I_{tsa} = 0 \quad [6\text{-}104]$$

$$nI_{tpb} + I_{tsb} = 0 \quad [6\text{-}105]$$

$$nI_{tpc} + I_{tsc} = 0 \quad [6\text{-}106]$$

$$R_T I_{tpa} - V_{tpa} + nV_{tsa} = 0 \quad [6\text{-}107]$$

$$R_T I_{tpb} - V_{tpb} + nV_{tsb} = 0 \quad [6\text{-}108]$$

$$R_T I_{tpc} - V_{tpc} + nV_{tsc} = 0 \quad [6\text{-}109]$$

$$V_{tpa} - V_{pag} + V_{png} = 0 \quad [6\text{-}110]$$

$$V_{tpb} - V_{pbg} + V_{png} = 0 \quad [6\text{-}111]$$

$$V_{tpc} - V_{pcg} + V_{png} = 0 \quad [6\text{-}112]$$

$$I_{sa} - I_{tsa} + I_{tsc} = 0 \quad [6\text{-}113]$$

$$I_{sb} - I_{tsb} + I_{tsa} = 0 \quad [6\text{-}114]$$

$$I_{sc} - I_{tsc} + I_{tsb} = 0 \quad [6\text{-}115]$$

$$V_{tsa} - V_{sag} + V_{sbg} = 0 \quad [6\text{-}116]$$

$$V_{tsb} - V_{sbg} + V_{scg} = 0 \quad [6\text{-}117]$$

$$V_{tsc} - V_{scg} + V_{sag} = 0 \quad [6\text{-}118]$$

While the equations above describe the relationships among the transformer terminals, the following equations are included to fully define the circuit shown in Figure 15. This figure includes a three-phase source with the neutral grounded with a resistor. It also includes a wye-connected load on the secondary system in addition to a grounding resistor connected to the load neutral conductor. Each of the load resistances may be different.

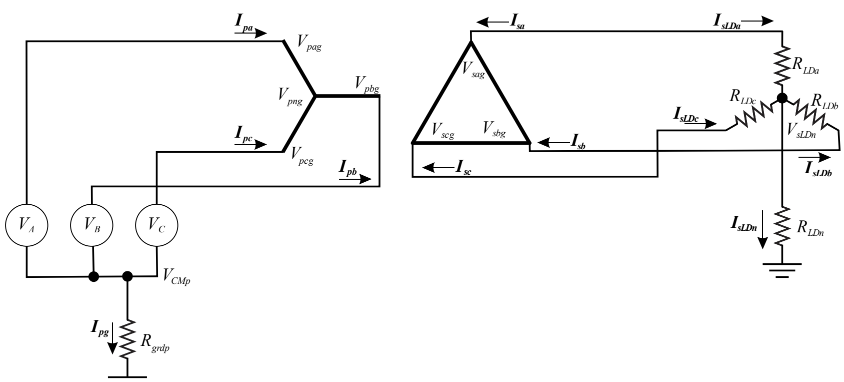

Figure 15: Wye-delta transformer with source and load

Assume a three-phase source (V_A, V_B and V_C are assumed to be sinusoidal and of the same frequency, but the equations and solution below only requires them to be ac)

$$V_{pag} - V_{CMp} = V_A \qquad [6\text{-}119]$$

$$V_{pbg} - V_{CMp} = V_B \qquad [6\text{-}120]$$

$$V_{pcg} - V_{CMp} = V_C \qquad [6\text{-}121]$$

Assume a grounding resistor at the source

$$R_{grdp} I_{pg} - V_{CMp} = 0 \qquad [6\text{-}122]$$

$$I_{pg} + I_{pa} + I_{pb} + I_{pc} = 0 \qquad [6\text{-}123]$$

$$I_{pa} + I_{pb} + I_{pc} = 0 \qquad [6\text{-}124]$$

Assume a wye connected load where each of the load resistances may be different

$$V_{sag} - V_{sLDn} - I_{sLDa} R_{LDa} = 0 \qquad [6\text{-}125]$$

$$V_{sbg} - V_{sLDn} - I_{sLDb} R_{LDb} = 0 \qquad [6\text{-}126]$$

$$V_{scg} - V_{sLDn} - I_{sLDc} R_{LDc} = 0 \qquad [6\text{-}127]$$

$$- I_{sLDn} + I_{sLDa} + I_{sLDb} + I_{sLDc} = 0 \qquad [6\text{-}128]$$

Account for the grounding resistor

$$V_{sLDn} - I_{sLDn} R_{LDn} = 0 \qquad [6\text{-}129]$$

Relate the secondary currents

$$I_{sa} + I_{sLDa} = 0 \qquad [6\text{-}130]$$

$$I_{sb} + I_{sLDb} = 0 \qquad [6\text{-}131]$$

$$I_{sc} + I_{sLDca} = 0 \qquad [6\text{-}132]$$

Solving this set of equations results in

$$I_{pa} = \frac{\left((-2R_{LDc}-2R_{LDb}+R_{LDa})V_C+(-2R_{LDc}+R_{LDb}-2R_{LDa})V_B+(4R_{LDc}+R_{LDb}+R_{LDa})V_A\right)n^2+R_T(-V_C-V_B+2V_A)}{(R_{LDa}(9R_{LDc}+9R_{LDb})+9R_{LDb}R_{LDc})n^4+(6R_{LDc}+6R_{LDb}+6R_{LDa})R_T n^2+3{R_T}^2} \qquad [6\text{-}1]$$

$$I_{pb} = -\frac{((-R_{LDc}+2R_{LDb}+2R_{LDa})V_C+(-R_{LDc}-R_{LDb}-4R_{LDa})V_B+(2R_{LDc}-R_{LDb}+2R_{LDa})V_A)n^2+R_T(V_C-2V_B+V_A)}{(R_{LDa}(9R_{LDc}+9R_{LDb})+9R_{LDb}R_{LDc})n^4+(6R_{LDc}+6R_{LDb}+6R_{LDa})R_T n^2+3{R_T}^2} \qquad [6\text{-}2]$$

$$I_{pc} = \frac{((R_{LDc}+4R_{LDb}+R_{LDa})V_C+(R_{LDc}-2R_{LDb}-2R_{LDa})V_B+(-2R_{LDc}-2R_{LDb}+R_{LDa})V_A)n^2+R_T(2V_C-V_B-V_A)}{(R_{LDa}(9R_{LDc}+9R_{LDb})+9R_{LDb}R_{LDc})n^4+(6R_{LDc}+6R_{LDb}+6R_{LDa})R_T n^2+3{R_T}^2} \qquad [6\text{-}3]$$

$$I_{tpa} = \frac{((-2R_{LDc}-2R_{LDb}+R_{LDa})V_C+(-2R_{LDc}+R_{LDb}-2R_{LDa})V_B+(4R_{LDc}+R_{LDb}+R_{LDa})V_A)n^2+R_T(-V_C-V_B+2V_A)}{(R_{LDa}(9R_{LDc}+9R_{LDb})+9R_{LDb}R_{LDc})n^4+(6R_{LDc}+6R_{LDb}+6R_{LDa})R_T n^2+3{R_T}^2} \qquad [6\text{-}4]$$

$$I_{tpb} = -\frac{((-R_{LDc}+2R_{LDb}+2R_{LDa})V_C+(-R_{LDc}-R_{LDb}-4R_{LDa})V_B+(2R_{LDc}-R_{LDb}+2R_{LDa})V_A)n^2+R_T(V_C-2V_B+V_A)}{(R_{LDa}(9R_{LDc}+9R_{LDb})+9R_{LDb}R_{LDc})n^4+(6R_{LDc}+6R_{LDb}+6R_{LDa})R_T n^2+3{R_T}^2} \qquad [6\text{-}5]$$

$$I_{tpc} = \frac{((R_{LDc}+4R_{LDb}+R_{LDa})V_C+(R_{LDc}-2R_{LDb}-2R_{LDa})V_B+(-2R_{LDc}-2R_{LDb}+R_{LDa})V_A)n^2+R_T(2V_C-V_B-V_A)}{(R_{LDa}(9R_{LDc}+9R_{LDb})+9R_{LDb}R_{LDc})n^4+(6R_{LDc}+6R_{LDb}+6R_{LDa})R_T n^2+3{R_T}^2} \qquad [6\text{-}6]$$

$$V_{pag} = V_A \tag{6-7}$$

$$V_{pbg} = V_B \tag{6-8}$$

$$V_{pcg} = V_C \tag{6-9}$$

$$V_{tpa} = \frac{-V_C - V_B + 2V_A}{3} \tag{6-10}$$

$$V_{tpb} = \frac{-V_C + 2V_B - V_A}{3} \tag{6-11}$$

$$V_{tpc} = \frac{2V_C - V_B - V_A}{3} \tag{6-12}$$

$$I_{sa} = -\frac{((-R_{LDc}-2R_{LDb})V_C+(R_{LDb}-R_{LDc})V_B+(2R_{LDc}+R_{LDb})V_A)n^3+R_T(V_A-V_C)n}{(R_{LDa}(3R_{LDc}+3R_{LDb})+3R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_T n^2+{R_T}^2} \tag{6-13}$$

$$I_{sb} = \frac{((R_{LDa}-R_{LDc})V_C+(-R_{LDc}-2R_{LDa})V_B+(2R_{LDc}+R_{LDa})V_A)n^3+R_T(V_A-V_B)n}{(R_{LDa}(3R_{LDc}+3R_{LDb})+3R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_T n^2+{R_T}^2} \tag{6-14}$$

$$I_{sc} = -\frac{((2R_{LDb}+R_{LDa})V_C+(-R_{LDb}-2R_{LDa})V_B+(R_{LDa}-R_{LDb})V_A)n^3+R_T(V_C-V_B)n}{(R_{LDa}(3R_{LDc}+3R_{LDb})+3R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_T n^2+{R_T}^2} \tag{6-15}$$

$$I_{tsa} = \frac{((2R_{LD}\ \ +2R_{LDb}-R_{LDa})V_C+(2R_{LDc}-R_{LDb}+2R_{LDa})V_B-(4R_{LDc}+R_{LDb}+R_{LDa})V_A)n^3+R_T(V_C+V_B-2V_A)n}{(R_{LDa}(9R_{LDc}+9R_{LDb})+9R_{LDb}R_{LDc})n^4+(6R_{LDc}+6R_{LDb}+6R_{LDa})R_T n^2+3{R_T}^2} \tag{6-16}$$

$$I_{tsb} = \frac{((-R_{LDc}+2R_{LDb}+2R_{LDa})V_C-(R_{LDc}+R_{LDb}+4R_{LDa})V_B+(2R_{LDc}-R_{LDb}+2R_{LDa})V_A)n^3+R_T(V_C-2V_B+V_A)n}{(R_{LDa}(9R_{LDc}+9R_{LDb})+9R_{LDb}R_{LDc})n^4+(6R_{LDc}+6R_{LDb}+6R_{LDa})R_T n^2+3{R_T}^2} \tag{6-17}$$

$$I_{tsc} = \frac{(-(R_{LDc}+4R_{LDb}+R_{LDa})V_C+(-R_{LDc}+2R_{LDb}+2R_{LDa})V_B+(2R_{LDc}+2R_{LDb}-R_{LDa})V_A)n^3+R_T(-V_C+V_B+V_A)n}{(R_{LDa}(9R_{LDc}+9R_{LDb})+9R_{LDb}R_{LDc})n^4+(6R_{LDc}+6R_{LDb}+6R_{LDa})R_T n^2+3{R_T}^2} \tag{6-18}$$

$$V_{sag} = \frac{R_{LDa}((-R_{LDc}-2R_{LDb})V_C+(R_{LDb}-R_{LDc})V_B+(2R_{LDc}+R_{LDb})V_A)n^3+R_T R_{LDa}(V_A-V_C)n}{(R_{LDa}(3R_{LDc}+3R_{LDb})+3R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_T n^2+{R_T}^2} \tag{6-19}$$

$$V_{sbg} = \frac{R_{LDb}((-R_{LDa}+R_{LDc})V_C+(R_{LDc}+2R_{LDa})V_B+(-2R_{LDc}-R_{LDa})V_A)n^3+R_T R_{LDb}(V_B-V_A)n}{(R_{LDa}(3R_{LDc}+3R_{LDb})+3R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_T n^2+{R_T}^2} \tag{6-20}$$

$$V_{scg} = \frac{R_{LDc}((2R_{LDb}+R_{LDa})V_C+(-R_{LDb}-2R_{LDa})V_B+(R_{LDa}-R_{LDb})V_A)n^3+R_T R_{LDc}(V_C-V_B)n}{(R_{LDa}(3R_{LDc}+3R_{LDb})+3R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_T n^2+{R_T}^2} \tag{6-21}$$

$$V_{tsa} = \frac{((R_{LDa}(-R_{LDc}-R_{LDb})-R_{LDb}R_{LDc})V_C+(R_{LDa}(-R_{LDc}-R_{LDb})-R_{LDb}R_{LDc})V_B+(R_{LDa}(2R_{LDc}+2R_{LDb})+2R_{LDb}R_{LDc})V_A)n^3}{(R_{LDa}(3R_{LDc}+3R_{LDb})+3R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_T n^2+{R_T}^2} + \frac{R_T(-R_{LDa})V_C-R_{LDb}V_B+(R_{LDb}+R_{LDa})V_A)n}{(R_{LDa}(3R_{LDc}+3R_{LDb})+3R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_T n^2+{R_T}^2} \tag{6-22}$$

$$V_{tsb} = -\frac{((R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})V_C+(R_{LDa}(-2R_{LDc}-2R_{LDb})-2R_{LDb}R_{LDc})V_B+(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})V_A)n^3}{(R_{LDa}(3R_{LDc}+3R_{LDb})+3R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_T n^2+{R_T}^2} - \frac{R_T(R_{LDc}V_C+(-R_{LDc}-R_{LDb})V_B+R_{LDb}V_A)n}{(R_{LDa}(3R_{LDc}+3R_{LDb})+3R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_T n^2+{R_T}^2} \tag{6-23}$$

$$V_{tsc} = -\frac{(R_{LDa}(-2R_{LDc}-2R_{LDb})-2R_{LDb}R_{LDc})V_C+(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})V_B+(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})V_A)n^3}{(R_{LDa}(3R_{LDc}+3R_{LDb})+3R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_T n^2+{R_T}^2} - \frac{R_T((-R_{LDc}-R_{LDa})V_C+R_{LDc}V_B+R_{LDa}V_A)n}{(R_{LDa}(3R_{LDc}+3R_{LDb})+3R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_T n^2+{R_T}^2} \tag{6-24}$$

$$V_{CMp} = 0 \tag{6-25}$$

$$I_{pg} = 0 \tag{6-26}$$

$$V_{png} = \frac{V_C + V_B + V_A}{3} \tag{6-27}$$

$$V_{sLDn} = 0 \qquad [6\text{-}28]$$

$$I_{sLDn} = 0 \qquad [6\text{-}29]$$

$$I_{sLDa} = \frac{((-R_{LDc}-2R_{LDb})V_C+(R_{LDb}-R_{LDc})V_B+(2R_{LDc}+R_{LDb})V_A)n^3+R_T(V_A-V_C)n}{(R_{LDa}(3R_{LDc}+3R_{LDb})+3R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_T n^2+{R_T}^2} \qquad [6\text{-}30]$$

$$I_{sLDb} = -\frac{((R_{LDa}-R_{LDc})V_C+(-R_{LDc}-2R_{LDa})V_B+(2R_{LDc}+R_{LDa})V_A)n^3+R_T(V_A-V_B)n}{(R_{LDa}(3R_{LDc}+3R_{LDb})+3R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_T n^2+{R_T}^2} \qquad [6\text{-}31]$$

$$I_{sLDc} = \frac{((2R_{LDb}+R_{LDa})V_C+(-R_{LDb}-2R_{LDa})V_B+(R_{LDa}-R_{LDb})V_A)n^3+R_T(V_C-V_B)n}{(R_{LDa}(3R_{LDc}+3R_{LDb})+3R_{LDb}R_{LDc})n^4+(2R_{LDc}+2R_{LDb}+2R_{LDa})R_T n^2+{R_T}^2} \qquad [6\text{-}32]$$

As expected, no current flows through the hull and the common mode voltage on the secondary is zero. The common mode voltage on the primary is equal to the common mode voltage of the sources.

If we make the assumption that R_T is negligible, then several of the solutions may be simplified:

$$I_{pa} = \frac{((-2R_{LDc}-2R_{LDb}+R_{LDa})V_C+(-2R_{LDc}+R_{LDb}-2R_{LDa})V_B+(4R_{LDc}+R_{LDb}+R_{LDa})V_A)}{(R_{LDa}(9R_{LDc}+9R_{LDb})+9R_{LDb}R_{LDc})n^2} \qquad [6\text{-}1a]$$

$$I_{pb} = -\frac{((-R_{LDc}+2R_{LDb}+2R_{LDa})V_C+(-R_{LDc}-R_{LDb}-4R_{LDa})V_B+(2R_{LDc}-R_{LDb}+2R_{LDa})V_A)}{(R_{LDa}(9R_{LDc}+9R_{LDb})+9R_{LDb}R_{LDc})n^2} \qquad [6\text{-}2a]$$

$$I_{pc} = \frac{((R_{LDc}+4R_{LDb}+R_{LDa})V_C+(R_{LDc}-2R_{LDb}-2R_{LDa})V_B+(-2R_{LDc}-2R_{LDb}+R_{LDa})V_A)}{(R_{LDa}(9R_{LDc}+9R_{LDb})+9R_{LDb}R_{LDc})n^2} \qquad [6\text{-}3a]$$

$$I_{tpa} = \frac{((-2R_{LDc}-2R_{LDb}+R_{LDa})V_C+(-2R_{LDc}+R_{LDb}-2R_{LDa})V_B+(4R_{LDc}+R_{LDb}+R_{LDa})V_A)}{(R_{LDa}(9R_{LDc}+9R_{LDb})+9R_{LDb}R_{LDc})n^2} \qquad [6\text{-}4a]$$

$$I_{tpb} = \frac{((R_{LDc}-2R_{LDb}-2R_{LDa})V_C+(R_{LDc}+R_{LDb}+4R_{LDa})V_B+(-2R_{LDc}+R_{LDb}-2R_{LDa})V_A)}{(R_{LDa}(9R_{LDc}+9R_{LDb})+9R_{LDb}R_{LDc})n^2} \qquad [6\text{-}5a]$$

$$I_{tpc} = \frac{((R_{LDc}+4R_{LDb}+R_{LDa})V_C+(R_{LDc}-2R_{LDb}-2R_{LDa})V_B+(-2R_{LDc}-2R_{LDb}+R_{LDa})V_A)}{(R_{LDa}(9R_{LDc}+9R_{LDb})+9R_{LDb}R_{LDc})n^2} \qquad [6\text{-}6a]$$

$$I_{sa} = -\frac{((-R_{LDc}-2R_{LDb})V_C+(R_{LDb}-R_{LDc})V_B+(2R_{LDc}+R_{LDb})V_A)}{(R_{LDa}(3R_{LD}\ \ +3R_{LDb})+3R_{LDb}R_{LDc})n} \qquad [6\text{-}13a]$$

$$I_{sb} = \frac{((R_{LDa}-R_{LDc})V_C+(-R_{LDc}-2R_{LDa})V_B+(2R_{LDc}+R_{LDa})V_A)}{(R_{LDa}(3R_{LDc}+3R_{LDb})+3R_{LDb}R_{LDc})n} \qquad [6\text{-}14a]$$

$$I_{sc} = -\frac{((2R_{LDb}+R_{LDa})V_C+(-R_{LDb}-2R_{LDa})V_B+(R_{LDa}-R_{LDb})V_A)}{(R_{LDa}(3R_{LDc}+3R_{LDb})+3R_{LDb}R_{LDc})n} \qquad [6\text{-}15a]$$

$$I_{tsa} = \frac{((2R_{LDc}+2R_{LDb}-R_{LDa})V_C+(2R_{LDc}-R_{LDb}+2R_{LDa})V_B-(4R_{LDc}+R_{LDb}+R_{LDa})V_A)}{(R_{LDa}(9R_{LDc}+9R_{LDb})+9R_{LDb}R_{LDc})n} \qquad [6\text{-}16a]$$

$$I_{tsb} = \frac{((-R_{LDc}+2R_{LDb}+2R_{LDa})V_C-(R_{LDc}+R_{LDb}+4R_{LDa})V_B+(2R_{LDc}-R_{LDb}+2R_{LDa})V_A)}{(R_{LDa}(9R_{LDc}+9R_{LDb})+9R_{LDb}R_{LDc})n} \qquad [6\text{-}17a]$$

$$I_{tsc} = \frac{(-(R_{LDc}+4R_{LDb}+R_{LDa})V_C+(-R_{LDc}+2R_{LDb}+2R_{LDa})V_B+(2R_{LDc}+2R_{LDb}-R_{LDa})V_A)}{(R_{LDa}(9R_{LDc}+9R_{LDb})+9R_{LDb}R_{LDc})n} \qquad [6\text{-}18a]$$

$$V_{sag} = \frac{R_{LDa}((-R_{LDc}-2R_{LDb})V_C+(R_{LDb}-R_{LDc})V_B+(2R_{LDc}+R_{LDb})V_A)}{(R_{LDa}(3R_{LDc}+3R_{LDb})+3R_{LDb}R_{LDc})n} \qquad [6\text{-}19a]$$

$$V_{sbg} = \frac{R_{LDb}((-R_{LDa}+R_{LDc})V_C+(R_{LDc}+2R_{LDa})V_B+(-2R_{LDc}-R_{LDa})V_A)}{(R_{LDa}(3R_{LDc}+3R_{LDb})+3R_{LDb}R_{LDc})n} \qquad [6\text{-}20a]$$

$$V_{scg} = \frac{R_{LDc}((2R_{LDb}+R_{LDa})V_C+(-R_{LDb}-2R_{LDa})V_B+(R_{LDa}-R_{LDb})V_A))}{(R_{LDa}(3R_{LDc}+3R_{LDb})+3R_{LDb}R_{LDc})n} \quad [6\text{-}21a]$$

$$V_{tsa} = \frac{((R_{LDa}(-R_{LDc}-R_{LDb})-R_{LDb}R_{LDc})V_C+(R_{LDa}(-R_{LDc}-R_{LDb})-R_{LDb}R_{LDc})V_B+(R_{LDa}(2R_{LDc}+2R_{LDb})+2R_{LDb}R_{LDc})V_A)}{(R_{LDa}(3R_{LDc}+3R_{LDb})+3R_{LDb}R_{LDc})n} \quad [6\text{-}22a]$$

$$V_{tsb} = -\frac{((R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})V_C+(R_{LDa}(-2R_{LDc}-2R_{LDb})-2R_{LDb}R_{LDc})V_B+(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})V_A)}{(R_{LDa}(3R_{LDc}+3R_{LDb})+3R_{LDb}R_{LDc})n} \quad [6\text{-}23a]$$

$$V_{tsc} = -\frac{(R_{LDa}(-2R_{LDc}-2R_{LDb})-2R_{LDb}R_{LDc})V_C+(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})V_B+(R_{LDa}(R_{LDc}+R_{LDb})+R_{LDb}R_{LDc})V_A)}{(R_{LDa}(3R_{LDc}+3R_{LDb})+3R_{LDb}R_{LDc})n} \quad [6\text{-}24a]$$

$$I_{sLDa} = \frac{((-R_{LDc}-2R_{LDb})V_C+(R_{LDb}-R_{LDc})V_B+(2R_{LDc}+R_{LDb})V_A)}{(R_{LDa}(3R_{LDc}+3R_{LDb})+3R_{LDb}R_{LDc})n} \quad [6\text{-}30a]$$

$$I_{sLDb} = -\frac{((R_{LDa}-R_{LDc})V_C+(-R_{LDc}-2R_{LDa})V_B+(2R_{LDc}+R_{LDa})V_A)}{(R_{LDa}(3R_{LDc}+3R_{LDb})+3R_{LDb}R_{LDc})n} \quad [6\text{-}31a]$$

$$I_{sLDc} = \frac{((2R_{LDb}+R_{LDa})V_C+(-R_{LDb}-2R_{LDa})V_B+(R_{LDa}-R_{LDb})V_A)}{(R_{LDa}(3R_{LDc}+3R_{LDb})+3R_{LDb}R_{LDc})n} \quad [6\text{-}32a]$$

If the sources are balanced (equal magnitude and phase shifted by 120˚) : then $V_A + V_B + V_C = 0$

$$I_{pa} = \frac{((-2R_{LDc}-2R_{LDb}+R_{LDa})V_C+(-2R_{LDc}+R_{LDb}-2R_{LDa})V_B+(4R_{LDc}+R_{LDb}+R_{LDa})V_A)}{(R_{LDa}(9R_{LDc}+9R_{LDb})+9R_{LDb}R_{LDc})n^2} \quad [6\text{-}1a]$$

$$I_{pa} = \frac{((-2R_{LDc}-2R_{LDb}+R_{LDa})(-V_A-V_B)+(-2R_{LDc}+R_{LDb}-2R_{LDa})V_B+(4R_{LDc}+R_{LDb}+R_{LDa})V_A)}{(R_{LDa}(9R_{LDc}+9R_{LDb})+9R_{LDb}R_{LDc})n^2} \quad [6\text{-}1aa]$$

$$I_{pa} = \frac{((3R_{LDb}-3R_{LDa})V_B+(6R_{LDc}+3R_{LDb})V_A)}{(R_{LDa}(9R_{LDc}+9R_{LDb})+9R_{LDb}R_{LDc})n^2} \quad [6\text{-}1ab]$$

The other phase currents will be shifted as well, based on the differences in the load resistances. This leads to the conclusions for unbalanced loads:

a) Because the phase current will have a component that is proportional to one of the other phase voltages, the current will not be in phase with the voltage. This implies a power factor of less than one.
b) Because the coefficient of the other phase voltage for the phase currents are a function of the load resistances, the phase currents will not be 120˚ apart.

If the sources are balanced, then the common mode voltage between the source neutral connection and the neutral of the three phase voltages is equal to zero. Hence the common mode voltage of the primary windings is zero.

If we further simplify and assume all of the load resistances are equal to R_{LD} and the sources are balanced then

$$I_{pa} = \frac{V_A}{(3R_{LD})n^2} \quad [6\text{-}1b]$$

$$I_{pb} = \frac{V_B}{(3R_{LD})n^2} \quad [6\text{-}2b]$$

$$I_{pc} = \frac{V_C}{(3R_{LD})n^2} \quad [6\text{-}3b]$$

$$I_{tpa} = \frac{V_A}{(3R_{LD})n^2} \quad [6\text{-}4b]$$

$$I_{tpb} = \frac{V_B}{(3R_{LD})n^2} \quad [6\text{-}5b]$$

$$I_{tpc} = \frac{V_C}{(3R_{LD})n^2} \quad [6\text{-}6b]$$

$$V_{tpa} = V_A \quad [6\text{-}10b]$$

$$V_{tpb} = V_B \quad [6\text{-}11b]$$

$$V_{tpc} = V_C \quad [6\text{-}12b]$$

$$I_{sa} = \frac{(V_C - V_A)}{3R_{LD}n} \quad [6\text{-}13b]$$

$$I_{sb} = \frac{(V_A - V_B)}{3R_{LD}n} \quad [6\text{-}14b]$$

$$I_{sc} = \frac{(V_B - V_C)}{3R_{LD}n} \quad [6\text{-}15b]$$

$$I_{tsa} = \frac{-V_A}{3R_{LD}n} \quad [6\text{-}16b]$$

$$I_{tsb} = \frac{-V_B}{3R_{LD}n} \quad [6\text{-}17b]$$

$$I_{tsc} = \frac{-V_C}{3R_{LD}n} \quad [6\text{-}18b]$$

$$V_{sag} = \frac{(V_A - V_C)}{3n} \quad [6\text{-}19b]$$

$$V_{sbg} = \frac{(V_B - V_A)}{3n} \quad [6\text{-}20b]$$

$$V_{scg} = \frac{V_C - V_B}{3n} \quad [6\text{-}21b]$$

$$V_{tsa} = \frac{V_A}{n} \quad [6\text{-}22b]$$

$$V_{tsb} = \frac{V_B}{n} \quad [6\text{-}23b]$$

$$V_{tsc} = \frac{V_C}{n} \quad [6\text{-}24b]$$

$$V_{png} = 0 \quad [6\text{-}27b]$$

$$I_{sLDa} = \frac{(V_A - V_C)}{3nR_{LD}} \quad [6\text{-}30b]$$

$$I_{sLDb} = \frac{(V_B - V_A)}{3nR_{LD}} \quad [6\text{-}31b]$$

$$I_{sLDc} = \frac{(V_C - V_B)}{3nR_{LD}} \quad [6\text{-}32b]$$

With balanced sources and a balanced load, the phase voltages and currents are in phase and thus the power factor will be 1.0. As expected, there is no current flowing through the hull, and the common mode voltages of the primary and secondary are zero.

Figure 16 depicts a wye-delta transformer with a balanced load and a ground fault on phase a of the secondary. The following equations are modified:

$$V_{sag}-V_{sLDn}-I_{sLDa}R_{LD}=0 \qquad \text{[6-125c]}$$

$$V_{sbg}-V_{sLDn}-I_{sLDb}R_{LD}=0 \qquad \text{[6-126c]}$$

$$V_{scg}-V_{sLDn}-I_{sLDc}R_{LD}=0 \qquad \text{[6-127c]}$$

$$I_{sa}+I_{sLDa}+I_{sfault}=0 \qquad \text{[6-130c]}$$

$$V_{sag}=0 \qquad \text{[6-133c]}$$

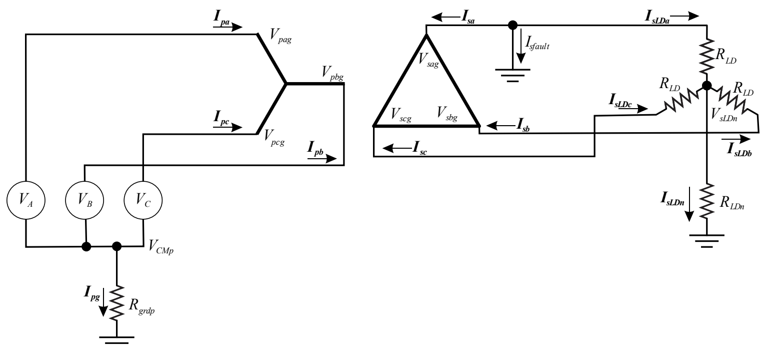

Figure 16: Wye-delta transformer with ground fault

Solving this set of equations results in:

$$I_{pa}=\frac{((-3R_{LD}R_{LDn}-4R_{LD}{}^2)V_C+(-3R_{LD}R_{LDn}-R_{LD}{}^2)V_B+(6R_{LD}R_{LDn}+5R_{LD}{}^2)V_A)n^2}{(27\ \ {}_{LD}{}^2R_{LDn}+9R_{LD}{}^3)n^4+(18\ \ {}_{LD}R_{LDn}+12R_{LD}{}^2)R_Tn^2+(3R_{LDn}+3R_{LD})R_T{}^2}+\frac{R_T((-R_{LDn}-R_{LD})V_C+(-R_{LDn}-R_{LD})V_B+(2R_{LDn}+2R_{LD})V_A)}{(27R_{LD}{}^2R_{LDn}+9R_{LD}{}^3)n^4+(18R_{LD}R_{LDn}+12R_{LD}{}^2)R_Tn^2+(3R_{LDn}+3R_{LD})R_T{}^2} \qquad \text{[6-1c]}$$

$$I_{pb}=-\frac{V_C-2V_B+V_A}{9R_{LD}n^2+3R_T} \qquad \text{[6-2c]}$$

$$I_{pc}=-\frac{\left((-6R_{LD}R_{LDn}-5R_{LD}{}^2)V_C+\left(3R_{LD}R_{LDn}+R_{LD}{}^2\right)V_B+\left(3R_{LD}R_{LDn}+4R_{LD}{}^2\right)V_A\right)n^2}{(27\ \ {}_{LD}{}^2R_{LDn}+9R_{LD}{}^3)n^4+(18\ \ {}_{LD}R_{LDn}+12R_{LD}{}^2)R_Tn^2+(3R_{LDn}+3R_{LD})R_T{}^2}-\frac{R_T\left((-2R_{LDn}-2R_{LD})VC\ \ (R_{LDn}+R_{LD})V_B+(R_{LD}\ \ +R_{LD})V_A\right)}{\left(27R_{LD}{}^2R_{LDn}+9R_{LD}{}^3\right)n^4+\left(18R_{LD}R_{LDn}+12R_{LD}{}^2\right)R_Tn^2+(3R_{LDn}+3R_{LD})R_T{}^2} \qquad \text{[6-3c]}$$

$$I_{tpa}=\frac{((-3R_{LD}R_{LDn}-4R_{LD}{}^2)V_C+(-3R_{LD}R_{LDn}-R_{LD}{}^2)V_B+(6R_{LD}R_{LDn}+5R_{LD}{}^2)V_A)n^2}{(27\ \ {}_{LD}{}^2R_{LDn}+9R_{LD}{}^3)n^4+(18\ \ {}_{LD}R_{LDn}+12R_{LD}{}^2)R_Tn^2+(3R_{LDn}+3R_{LD})R_T{}^2}+\frac{R_T((-R_{LDn}-R_{LD})VC+(-R_{LDn}-R_{LD})V_B+(2R_{LDn}+2R_{LD})V_A)}{(27R_{LD}{}^2R_{LDn}+9R_{LD}{}^3)n^4+(18R_{LD}R_{LDn}+12R_{LD}{}^2)R_Tn^2+(3R_{LDn}+3R_{LD})R_T{}^2} \qquad \text{[6-4c]}$$

$$I_{tpb}=-\frac{V_C-2V_B+V_A}{9R_{LD}n^2+3R_T} \qquad \text{[6-5c]}$$

$$I_{tpc}=-\frac{\left((-6R_{LD}R_{LDn}-5R_{LD}{}^2)V_C+\left(3R_{LD}R_{LDn}+R_{LD}{}^2\right)V_B+\left(3R_{LD}R_{LDn}+4R_{LD}{}^2\right)V_A\right)n^2}{\left(27R_{LD}{}^2R_{LDn}+9R_{LD}{}^3\right)n^4+\left(18R_{LD}R_{LDn}+12R_{LD}{}^2\right)R_Tn^2+(3R_{LDn}+3R_{LD})R_T{}^2}-\frac{R_T\left((-2R_{LDn}-2R_{LD})VC\ \ (R_{LDn}+R_{LD})V_B+(R_{LDn}+R_{LD})V_A\right)}{(27\ \ {}_{LD}{}^2R_{LDn}+9R_{LD}{}^3)n^4+\left(18R_{LD}R_{LDn}+12R_{LD}{}^2\right)R_Tn^2+(3R_{LDn}+3R_{LD})R_T{}^2} \qquad \text{[6-6c]}$$

$$V_{pag}=V_A \qquad \text{[6-7c]}$$

$$V_{pbg} = V_B \quad [6\text{-}8c]$$

$$V_{pcg} = V_C \quad [6\text{-}9c]$$

$$V_{tpa} = \frac{-V_C-V_B+2V_A}{3} \quad [6\text{-}10c]$$

$$V_{tpb} = -\frac{V_C-2V_B+V_A}{3} \quad [6\text{-}11c]$$

$$V_{tp} \ = -\frac{-2V_C+V_B+V_A}{3} \quad [6\text{-}12c]$$

$$I_{sa} = -\frac{((-R_{LDn}-R_{LD})V_C+(R_{LDn}+R_{LD})V_A)n}{(3R_{LD}R_{LDn}+R_{LD}{}^2)n^2+(R_{LDn}+R_{LD})R_T} \quad [6\text{-}13c]$$

$$I_{sb} = \frac{(-R_{LD}{}^2V_C+(-3R_{LD}R_{LDn}-R_{LD}{}^2)V_B+(3R_{LD}R_{LDn}+2R_{LD}{}^2)V_A)n^3+R_T((-R_{LDn}-R_{LD})V_B+(R_{LDn}+R_{LD})V_A)n}{(9R_{LD}{}^2R_{LDn}+3R_{LD}{}^3)n^4+(6R_{LD}R_{LDn}+4R_{LD}{}^2)R_Tn^2+(R_{LDn}+R_{LD})R_T{}^2} \quad [6\text{-}14c]$$

$$I_{sc} = \frac{((-3R_{LD}R_{LDn}-2R_{LD}{}^2)V_C+(3R_{LD}R_{LDn}+R_{LD}{}^2)V_B+R_{LD}{}^2V_A)n^3+R\ \ ((-R_{LDn}-R_{LD})V_C+(R_{LDn}+R_{LD})V_B)n}{(9R_{LD}{}^2R_{LDn}+3R_{LD}{}^3)n^4+(6R_{LD}R_{LDn}+4R_{LD}{}^2)R_Tn^2+(R_{LDn}+R_{LD})R_T{}^2} \quad [6\text{-}15c]$$

$$I_{tsa} = -\frac{\left((-3R_{LD}R_{LDn}-4R_{LD}{}^2)V_C+(-3R_{LD}R_{LDn}-R_{LD}{}^2)V_B+(6R_{LD}R_{LDn}+5R_{LD}{}^2)V_A\right)n^3}{(27\ \ {}_{LD}{}^2R_{LDn}+9R_{LD}{}^3)n^4+(18R_{LD}R_{LDn}+12R_{LD}{}^2)R_Tn^2+(3R_{LDn}+3R_{LD})R_T{}^2} - \frac{R_T((-R_{LDn}-R_{LD})V_C+(-R_{LDn}-R_{LD})V_B+(2R_{LDn}+2R_{LD})V_A)n}{(27R_{LD}{}^2R_{LDn}+9R_{LD}{}^3)n^4+(18R_{LD}R_{LDn}+12R_{LD}{}^2)R_Tn^2+(3R_{LDn}+3R_{LD})R_T{}^2} \quad [6\text{-}16c]$$

$$I_{tsb} = \frac{(V_C-2V_B+V_A)n}{9R_{LD}n^2+3R_T} \quad [6\text{-}17c]$$

$$I_{tsc} = \frac{((-6R_{LD}R_{LDn}-5R_{LD}{}^2)V_C+(3R_{LD}R_{LDn}+R_{LD}{}^2)V_B+(3R_{LD}R_{LDn}+4R_{LD}{}^2)V_A)n^3}{(27R_{LD}{}^2R_{LDn}+9R_{LD}{}^3)n^4+(18R_{LD}R_{LDn}+12R_{LD}{}^2)R_Tn^2+(3R_{LDn}+3R_{LD})R_T{}^2} + \frac{R_T((-2R_{LDn}-2R_{LD})V_C+(R_{LDn}+R_{LD})V_B+(R_{LDn}+R_{LD})V_A)n}{(27R_{LD}{}^2R_{LDn}+9R_{LD}{}^3)n^4+(18R_{LD}R_{LDn}+12R_{LD}{}^2)R_Tn^2+(3R_{LDn}+3R_{LD})R_T{}^2} \quad [6\text{-}18c]$$

$$V_{sag} = 0 \quad [6\text{-}19c]$$

$$V_{sbg} = -\frac{((-3R_{LD}{}^2R_{LDn}-R_{LD}{}^3V_C+(-3R_{LD}{}^2R_{LDn}-R_{LD}{}^3)V_B+(6R_{LD}{}^2R_{LDn}+2R_{LD}{}^3)V_A)n^3}{(9R_{LD}{}^2R_{LDn}+3R_{LD}{}^3)n^4+(6R_{LD}R_{LDn}+4R_{LD}{}^2)R_Tn^2+(R_{LDn}+R_{LD})R_T{}^2} - \frac{R_T(-R_{LD}R_{LDn}V_C+(-R_{LD}R_{LDn}-R_{LD}{}^2)V_B+(2R_{LD}R_{LDn}+R_{LD}{}^2)V_A)n}{(9R_{LD}{}^2R_{LDn}+3R_{LD}{}^3)n^4+(6R_{LD}R_{LDn}+4R_{LD}{}^2)R_Tn^2+(R_{LDn}+R_{LD})R_T{}^2} \quad [6\text{-}20c]$$

$$V_{scg} = -\frac{\left((-6R_{LD}{}^2R_{LDn}-2R_{LD}{}^3)V_C+(3R_{LD}{}^2R_{LDn}+R_{LD}{}^3)V_B+(3R_{LD}{}^2R_{LDn}+R_{LD}{}^3)V_A\right)n^3}{(9R_{LD}{}^2R_{LDn}+3R_{LD}{}^3)n^4+(6R_{LD}R_{LDn}+4R_{LD}{}^2)R_Tn^2+(R_{LDn}+R_{LD})R_T{}^2} - \frac{R_T((-2R_{LD}R_{LDn}-R_{LD}{}^2)V_C+(R_{LD}R_{LDn}+R_{LD}{}^2)V_B+R_{LD}R_{LDn}V_A)n}{(9R_{LD}{}^2R_{LDn}+3R_{LD}{}^3)n^4+(6R_{LD}R_{LDn}+4R_{LD}{}^2)R_Tn^2+(R_{LDn}+R_{LD})R_T{}^2} \quad [6\text{-}21c]$$

$$V_{tsa} = \frac{((-3R_{LD}{}^2R_{LDn}-R_{LD}{}^3)V_C+(-3R_{LD}{}^2R_{LDn}-R_{LD}{}^3)V_B+(6R_{LD}{}^2R_{LDn}+2R_{LD}{}^3)V_A)n^3}{(9R_{LD}{}^2R_{LDn}+3R_{LD}{}^3)n^4+(6R_{LD}R_{LDn}+4R_{LD}{}^2)R_Tn^2+(R_{LDn}+R_{LD})R_T{}^2} + \frac{R_T(-R_{LD}R_{LDn}V_C+(-R_{LD}R_{LDn}-R_{LD}{}^2)V_B+(2R_{LD}R_{LDn}+R_{LD}{}^2)V_A)n}{(9R_{LD}{}^2R_{LDn}+3R_{LD}{}^3)n^4+(6R_{LD}R_{LDn}+4R_{LD}{}^2)R_Tn^2+(R_{LDn}+R_{LD})R_T{}^2} \quad [6\text{-}22c]$$

$$V_{tsb} = -\frac{(R_{LD}V_C-2R_{LD}V_B+R_{LD}V_A)n}{3R_{LD}n^2+R_T} \quad [6\text{-}23c]$$

$$V_{tsc} = -\frac{\left((-6R_{LD}{}^2R_{LDn}-2R_{LD}{}^3)V_C+(3R_{LD}{}^2R_{LDn}+R_{LD}{}^3)V_B+(3R_{LD}{}^2R_{LDn}+R_{LD}{}^3)V_A\right)n^3}{(9R_{LD}{}^2R_{LDn}+3R_{LD}{}^3)n^4+(6R_{LD}R_{LDn}+4R_{LD}{}^2)R_Tn^2+(R_{LDn}+R_{LD})R_T{}^2} - \frac{R_T\left((-2R_{LD}R_{LDn}-R_{LD}{}^2)V_C+(R_{LD}R_{LDn}+R_{LD}{}^2)V_B+R_{LD}R_{LDn}V_A\right)n}{(9R_{LD}{}^2R_{LDn}+3R_{LD}{}^3)n^4+(6R_{LD}R_{LDn}+4R_{LD}{}^2)R_Tn^2+(R_{LDn}+R_{LD})R_T{}^2} \quad [6\text{-}24c]$$

$$V_{CMp} = 0 \tag{6-25c}$$

$$I_{pg} = 0 \tag{6-26c}$$

$$V_{png} = \frac{V_C+V_B+V_A}{3} \tag{6-27c}$$

$$V_{sLDn} = -\frac{(R_{LD}R_{LDn}V_A-R_{LD}R_{LDn}V_C)n}{(3R_{LD}R_{LDn}+R_{LD}{}^2)n^2+(R_{LDn}+R_{LD})R_T} \tag{6-28c}$$

$$I_{sLDn} = -\frac{(R_{LD}V_A-R_{LD}V_C)n}{(3R_{LD}R_{LDn}+R_{LD}{}^2)n^2+(R_{LDn}+R_{LD})R_T} \tag{6-29c}$$

$$I_{sLDa} = \frac{(R_{LDn}V_A-R_{LDn}V_C)n}{(3R_{LD}R_{LDn}+R_{LD}{}^2)n^2+(R_{LDn}+R_{LD})R_T} \tag{6-30c}$$

$$I_{sLDb} = -\frac{(-R_{LD}{}^2V_C+(-3R_{LD}R_{LDn}-R_{LD}{}^2)V_B+(3R_{LD}R_{LDn}+2R_{LD}{}^2)V_A)n^3+R_T((-R_{LDn}-R_{LD})V_B+(R_{LDn}+R_{LD})V_A)n}{(9R_{LD}{}^2R_{LDn}+3R_{LD}{}^3)n^4+(6R_{LD}R_{LDn}+4R_{LD}{}^2)R_Tn^2+(R_{LDn}+R_{LD})R_T{}^2} \tag{6-31c}$$

$$I_{sLDc} = -\frac{((-3R_{LD}R_{LDn}-2R_{LD}{}^2)V_C+(3R_{LD}R_{LDn}+R_{LD}{}^2)V_B+R_{LD}{}^2V_A)n^3+R_T((-R_{LDn}-R_{LD})V_C+(R_{LDn}+R_{LD})V_B)n}{(9R_{LD}{}^2R_{LDn}+3R_{LD}{}^3)n^4+(6R_{LD}R_{LDn}+4R_{LD}{}^2)R_Tn^2+(R_{LDn}+R_{LD})R_T{}^2} \tag{6-32c}$$

$$I_{sfault} = \frac{(R_{LD}V_A-R_{LD}V_C)n}{(3R_{LD}R_{LDn}+R_{LD}{}^2)n^2+(R_{LDn}+R_{LD})R_T} \tag{6-33c}$$

If we also assume R_T is negligible, then

$$I_{pa} = \frac{((-3R_{LDn}-4R_{LD})V_C+(-3R_{LDn}-R_{LD})V_B+(6R_{LDn}+5R_{LD})V_A)}{(27\ \ _{LD}R_{LDn}+9R_{LD}{}^2)n^2} \tag{6-1d}$$

$$I_{pb} = -\frac{V_C-2V_B+V_A}{9R_{LD}n^2} \tag{6-2d}$$

$$I_{pc} = \frac{((6R_{LDn}+5R_{LD})V_C+(-3R_{LDn}-R_{LD})V_B+(-3R_{LDn}-4R_{LD})V_A)}{(27\ \ _{LD}R_{LDn}+9R_{LD}{}^2)n^2} \tag{6-3d}$$

$$I_{tpa} = \frac{((-3R_{LDn}-4R_{LD})V_C+(-3R_{LDn}-R_{LD})V_B+(6R_{LDn}+5R_{LD})V_A)}{(27R_{LD}R_{LDn}+9R_{LD}{}^2)n^2} \tag{6-4d}$$

$$I_{tpb} = -\frac{V_C-2V_B+V_A}{9R_{LD}n^2} \tag{6-5d}$$

$$I_{tpc} = \frac{((6R_{LDn}+5R_{LD}{}^2)V_C+(-3R_{LDn}-R_{LD})V_B+(-3R_{LDn}-4R_{LD})V_A)}{(27R_{LD}R_{LDn}+9R_{LD}{}^2)n^2} \tag{6-6d}$$

$$I_{sa} = \frac{((R_{LDn}+R_{LD})V_C-(R_{LDn}+R_{LD})V_A)}{(3R_{LD}R_{LDn}+R_{LD}{}^2)n} \tag{6-13d}$$

$$I_{sb} = \frac{(-R_{LD}V_C+(-3R_{LDn}-R_{LD})V_B+(3R_{LDn}+2R_{LD})V_A)}{(9R_{LD}R_{LDn}+3R_{LD}{}^2)n} \tag{6-14d}$$

$$I_{sc} = \frac{((-3R_{LDn}-2R_{LD})V_C+(3R_{LDn}+R_{LD})V_B+R_{LD}V_A)}{(9R_{LD}R_{LDn}+3R_{LD}{}^2)n} \tag{6-15d}$$

$$I_{tsa} = -\frac{((-3R_{LD}\ \ -4R_{LD})V_C+(-3R_{LDn}-R_{LD})V_B+(6R_{LDn}+5R_{LD})V_A)}{(27R_{LD}R_{LDn}+9R_{LD}{}^2)n} \tag{6-16d}$$

$$I_{tsb} = \frac{(V_C-2V_B+V_A)}{9R_{LD}n} \tag{6-17d}$$

$$I_{tsc} = \frac{((-6R_{LDn}-5R_{LD})V_C+(3R_{LDn}+R_{LD})V_B+(3R_{LDn}+4R_{LD})V_A)}{(27R_{LD}R_{LDn}+9R_{LD}{}^2)n} \tag{6-18d}$$

$$V_{sag} = 0 \quad [6\text{-}19d]$$

$$V_{sbg} = -\frac{((-3R_{LDn}-R_{LD})V_C+(-3R_{LDn}-R_{LD})V_B+(6R_{LDn}+2R_{LD})V_A)}{(9R_{LDn}+3R_{LD})n} \quad [6\text{-}20d]$$

$$V_{scg} = -\frac{((-6R_{LDn}-2R_{LD})V_C+(3R_{LDn}+R_{LD})V_B+(3R_{LDn}+R_{LD})V_A)}{(9R_{LDn}+3R_{LD})n} \quad [6\text{-}21d]$$

$$V_{tsa} = \frac{((-3R_{LDn}-R_{LD})V_C+(-3R_{LDn}-R_{LD})V_B+(6R_{LDn}+2R_{LD})V_A)}{(9R_{LDn}+3R_{LD})n} \quad [6\text{-}22d]$$

$$V_{tsb} = -\frac{(R_{LD}V_C-2R_{LD}V_B+R_{LD}V_A)}{3R_{LD}n} \quad [6\text{-}23d]$$

$$V_{tsc} = -\frac{((-6R_{LDn}-2R_{LD})V_C+(3R_{LDn}+R_{LD})V_B+(3R_{LDn}+R_{LD})V_A)}{(9R_{LDn}+3R_{LD})n} \quad [6\text{-}24d]$$

$$V_{sLDn} = -\frac{(R_{LDn}V_A-R_{LDn}V_C)}{(3R_{LDn}+R_{LD})n} \quad [6\text{-}28d]$$

$$I_{sLDn} = -\frac{(V_A-V_C)}{(3R_{LDn}+R_{LD})n} \quad [6\text{-}29d]$$

$$I_{sLDa} = \frac{(R_{LDn}V_A-R_{LDn}V_C)}{(3R_{LD}R_{LDn}+R_{LD}{}^2)n} \quad [6\text{-}30d]$$

$$I_{sLDb} = -\frac{(-R_{LD}V_C+(-3R_{LDn}-R_{LD})V_B+(3R_{LDn}+2R_{LD})V_A)}{(9R_{LD}R_{LDn}+3R_{LD}{}^2)n} \quad [6\text{-}31d]$$

$$I_{sLDc} = -\frac{((-3R_{LDn}-2R_{LD})V_C+(3R_{LDn}+R_{LD})V_B+R_{LD}V_A)}{(9R_{LD}R_{LDn}+3R_{LD}{}^2)n} \quad [6\text{-}32d]$$

$$I_{sfault} = \frac{(V_A-V_C)}{(3R_{LDn}+R_{LD})n} \quad [6\text{-}33d]$$

If the sources are balanced then …

$$I_{pa} = \frac{((-R_{LD})V_C+(3R_{LDn}+2R_{LD})V_A)}{(9R_{LD}R_{LDn}+3R_{LD}{}^2)n^2} \quad [6\text{-}1e]$$

$$I_{pb} = \frac{V_B}{3R_{LD}n^2} \quad [6\text{-}2e]$$

$$I_{pc} = \frac{((3R_{LDn}+2R_{LD})V_C+(-R_{LD})V_A)}{(9R_{LD}R_{LDn}+3R_{LD}{}^2)n^2} \quad [6\text{-}3e]$$

$$I_{tpa} = \frac{((-R_{LD})V_C+(3R_{LDn}+2R_{LD})V_A)}{(9R_{LD}R_{LDn}+3R_{LD}{}^2)n^2} \quad [6\text{-}4e]$$

$$I_{tpb} = \frac{V_B}{3R_{LD}n^2} \quad [6\text{-}5e]$$

$$I_{tpc} = \frac{((3R_{LDn}+2R_{LD}{}^2)V_C+(-R_{LD})V_A)}{(9R_{LD}R_{LDn}+3R_{LD}{}^2)n^2} \quad [6\text{-}6e]$$

$$I_{sa} = \frac{((R_{LDn}+R_{LD})V_C-(R_{LDn}+R_{LD})V_A)}{(3R_{LD}R_{LDn}+R_{LD}{}^2)n} \quad [6\text{-}13e]$$

$$I_{sb} = \frac{((-R_{LDn})V_B+(R_{LDn}+R_{LD})V_A)}{(3R_{LD}R_{LDn}+R_{LD}{}^2)n} \quad [6\text{-}14e]$$

$$I_{sc} = \frac{((-R_{LDn}-R_{LD})V_C+(R_{LDn})V_B)}{(3R_{LD}R_{LDn}+R_{LD}{}^2)n} \quad [6\text{-}15e]$$

$$I_{tsa} = -\frac{((-R_{LD})V_C+(3R_{LDn}+2R_{LD})V_A)}{(9R_{LD}R_{LDn}+3R_{LD}{}^2)n} \quad [6\text{-}16e]$$

$$I_{tsb} = \frac{-V_B}{3R_{LD}n} \quad [6\text{-}17e]$$

$$I_{tsc} = \frac{((-3R_{LDn}-2R_{LD})V_C+(R_{LD})V_A)}{(9R_{LD}R_{LDn}+3R_{LD}{}^2)n} \quad [6\text{-}18e]$$

$$V_{sag} = 0 \quad [6\text{-}19e]$$

$$V_{sbg} = -\frac{V_A}{n} \quad [6\text{-}20e]$$

$$V_{scg} = \frac{V_C}{n} \quad [6\text{-}21e]$$

$$V_{tsa} = \frac{V_A}{n} \quad [6\text{-}22e]$$

$$V_{tsb} = \frac{V_B}{n} \quad [6\text{-}23e]$$

$$V_{tsc} = \frac{V_C}{n} \quad [6\text{-}24e]$$

$$I_{sLDa} = \frac{(R_{LDn}V_A-R_{LDn}V_C)}{(3R_{LD}R_{LDn}+R_{LD}{}^2)n} \quad [6\text{-}30e]$$

$$I_{sLDb} = -\frac{((-R_{LDn})V_B+(R_{LDn}+R_{LD})V_A)}{(3R_{LD}R_{LDn}+R_{LD}{}^2)n} \quad [6\text{-}31e]$$

$$I_{sLDc} = -\frac{((-R_{LDn}-R_{LD})V_C+(R_{LDn})V_B)}{(3R_{LD}R_{LDn}+R_{LD}{}^2)n} \quad [6\text{-}32e]$$

If $R_{LDn} \gg R_{LD}$ then

$$I_{pa} = \frac{V_A}{3R_{LD}n^2} \quad [6\text{-}1f]$$

$$I_{pb} = \frac{V_B}{3R_{LD}n^2} \quad [6\text{-}2f]$$

$$I_{pc} = \frac{V_C}{3R_{LD}n^2} \quad [6\text{-}3f]$$

$$I_{tpa} = \frac{V_A}{3R_{LD}n^2} \quad [6\text{-}4f]$$

$$I_{tpb} = \frac{V_B}{3R_{LD}n^2} \quad [6\text{-}5f]$$

$$I_{tpc} = \frac{V_C}{3R_{LD}n^2} \quad [6\text{-}6f]$$

$$I_{sa} = \frac{(V_C-V_A)}{3R_{LD}n} \quad [6\text{-}13f]$$

$$I_{sb} = \frac{(V_A-V_B)}{3R_{LD}n} \quad [6\text{-}14f]$$

$$I_{sc} = \frac{(V_B-V_C)}{3R_{LD}n} \quad [6\text{-}15f]$$

$$I_{tsa} = -\frac{V_A}{3R_{LD}n} \quad [6\text{-}16f]$$

$$I_{tsb} = -\frac{V_B}{3R_{LD}n} \quad [6\text{-}17f]$$

$$I_{tsc} = -\frac{V_C}{3R_{LD}n} \quad [6\text{-}18f]$$

$$I_{sLDa} = \frac{(V_A - V_C)}{3R_{LD}n} \quad [6\text{-}30e]$$

$$I_{sLDb} = \frac{(V_B - V_A)}{3R_{LD}n} \quad [6\text{-}31e]$$

$$I_{sLDc} = \frac{(V_C - V_B)}{3R_{LD}n} \quad [6\text{-}32e]$$

Conclusions from this analysis includes:

a) In a ground fault, the secondary grounding resistor results in the primary currents not being 120˚ apart
b) If the secondary is not grounded, then the ground fault has no impact on the primary and secondary currents.

7. Multi-Phase Transformers

Shipboard three phase transformers are not always constructed in the form of three single phase transformers. In some cases, all six windings (three primary and three secondary) are on the same core. The configuration of this core can also vary. Additionally, especially for transformers supplying variable frequency drives (VFDs), in addition to a delta connected primary, there may be two sets of secondary windings: one set configured in wye, and the other in delta. This configuration is used as part of a 12-pulse rectifier designed to keep harmonic currents within specifications.

To analyze these transformers, this chapter will model the magnetic circuit consisting of the core and the windings. To simplify analysis, the reluctance (R_m) of the core will be assumed to be very small and therefore may be neglected. First, we will analyze a single-phase transformer to demonstrate it yields the expected relationships.

Figure 17 depicts a single-phase transformer. In the magnetic circuit, F_{pa} and F_{sa} represent the magnetomotive force which is analogous to a voltage in an electrical circuit. The magnetic flux is represented by Φ_a and is analogous to a current in an electrical circuit.

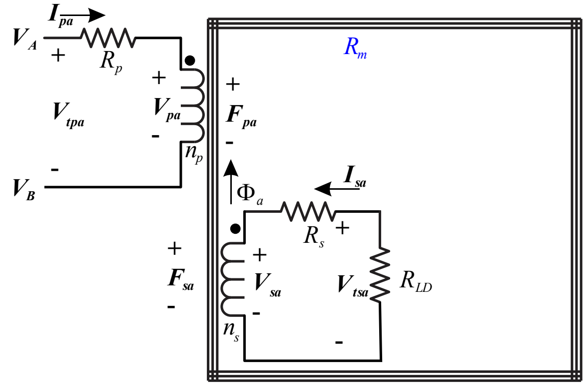

Figure 17: Single Phase Transformer – magnetic circuit

The voltage across each of the coils is related to the flux:

$$V_{pa} = n_p \frac{d\Phi_a}{dt} \tag{7-201}$$

$$V_{sa} = n_s \frac{d\Phi_a}{dt} \tag{7-202}$$

The current into the coil is related to the magnetomotive force:

$$F_{pa} = n_p I_{pa} \tag{7-203}$$

$$F_{sa} = n_s I_{sa} \tag{7-204}$$

The magnetic circuit equation is given by:

$$F_{sa} + F_{pa} = n_s I_{sa} + n_p I_{pa} = 0 \tag{7-205}$$

Dividing equation 7-201 by 7-202 results in:

$$V_{pa} = \frac{n_p}{n_s} V_{sa} \tag{7-206}$$

Equation 7-205

$$I_{pa} = -\frac{n_s}{n_p} I_{sa} \quad [7\text{-}207]$$

Define

$$n = \frac{n_p}{n_s} \quad [7\text{-}208]$$

Substitute into equations 7-206 and 7-207 results in the classic ideal transformer equations.

$$V_{pa} = nV_{sa} \quad [7\text{-}209]$$

$$I_{pa} = -\frac{I_{sa}}{n} \quad [7\text{-}210]$$

Figure 18 depicts a shell-type three-phase transformer. The two end limbs provide a flux return path for each of the three center main limbs. Each main limb behaves essentially as an independent single-phase transformer. All of the equations from chapters 3-6 apply to this type of transformer.

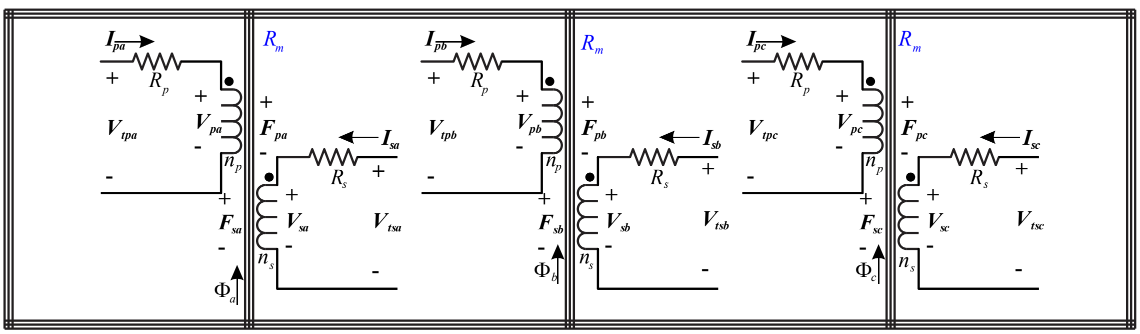

Figure 18: Shell-type three-phase transformer

Figure 19 depicts the more common core-type transformer. The flux return path for each main leg is through the other main legs. It is not obvious that the three sets of windings are independent of each other. The magnetomotive force between the top and bottom of a leg is defined as F_l. Since we are neglecting the reluctance of the magnetic circuit, F_l is the same for each leg. In the shell type transformer and in the single phase transformer, a flux return path is provided that forces F_l to be zero. If F_l in the core transformer is found to be zero, then the magnetic circuit for each leg is equivalent to the shell-type transformer or from a bank of three single-phase transformers. If F_l is not zero, then the magnetic circuits of the three legs are coupled; they should not be treated as independent transformers.

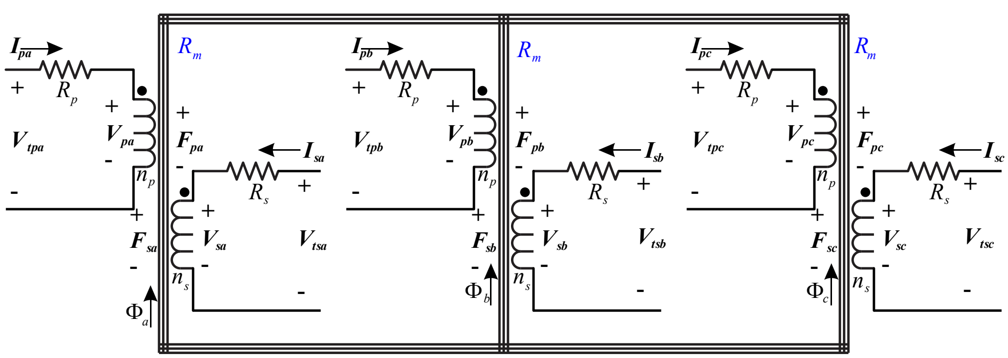

Figure 19: Core-type three-phase transformer

Neglecting the magnetic reluctance (R_m), the equations for this transformer are given by:

$$V_{pa} = n_p \frac{d\Phi_a}{dt} \quad [7\text{-}101]$$

$$V_{pb} = n_p \frac{d\Phi_b}{dt} \quad [7\text{-}102]$$

$$V_{pc} = n_p \frac{d\Phi_c}{dt} \quad [7\text{-}103]$$

$$V_{sa} = n_s \frac{d\Phi_a}{dt} \quad [7\text{-}104]$$

$$V_{sb} = n_s \frac{d\Phi_b}{dt} \quad [7\text{-}105]$$

$$V_{sc} = n_s \frac{d\Phi_c}{dt} \quad [7\text{-}106]$$

$$F_{pa} = n_p I_{pa} \quad [7\text{-}107]$$

$$F_{pb} = n_p I_{pb} \quad [7\text{-}108]$$

$$F_{pc} = n_p I_{pc} \quad [7\text{-}109]$$

$$F_{sa} = n_s I_{sa} \quad [7\text{-}110]$$

$$F_{sb} = n_s I_{sb} \quad [7\text{-}111]$$

$$F_{sc} = n_s I_{sc} \quad [7\text{-}112]$$

$$F_1 = F_{pa} + F_{sa} \quad [7\text{-}113]$$

$$F_1 = F_{pb} + F_{sb} \quad [7\text{-}114]$$

$$F_1 = F_{pc} + F_{sc} \quad [7\text{-}115]$$

$$\Phi_a + \Phi_b + \Phi_c = 0 \quad [7\text{-}116]$$

Add series resistances

$$V_{pa} = V_{tpa} - R_p I_{pa} \quad [7\text{-}117]$$

$$V_{pb} = V_{tpb} - R_p I_{pb} \quad [7\text{-}118]$$

$$V_{pc} = V_{tpc} - R_p I_{pc} \quad [7\text{-}119]$$

$$V_{sa} = V_{tsa} - R_s I_{sa} \quad [7\text{-}120]$$

$$V_{sb} = V_{tsb} - R_s I_{sb} \quad [7\text{-}121]$$

$$V_{sc} = V_{tsc} - R_s I_{sc} \quad [7\text{-}122]$$

Assume the flux is sinusoidal

$$\frac{d\Phi_a}{dt} = \omega \hat{J} \Phi_a \quad [7\text{-}211]$$

$$\frac{d\Phi_b}{dt} = \omega \hat{J} \Phi_b \quad [7\text{-}212]$$

$$\frac{d\Phi_c}{dt} = \omega\hat{J}\Phi_c \qquad [7\text{-}213]$$

We can eliminate the derivatives through direct substitution …

$$V_{pa} = n_p\omega\hat{J}\Phi_a \qquad [7\text{-}101a]$$

$$V_{pb} = n_p\omega\hat{J}\Phi_b \qquad [7\text{-}102a]$$

$$V_{pc} = n_p\omega\hat{J}\Phi_c \qquad [7\text{-}103a]$$

$$V_{sa} = n_s\omega\hat{J}\Phi_a \qquad [7\text{-}104a]$$

$$V_{sb} = n_s\omega\hat{J}\Phi_b \qquad [7\text{-}105a]$$

$$V_{sc} = n_s\omega\hat{J}\Phi_c \qquad [7\text{-}106a]$$

This system of equations can be solved in terms of the transformer terminal voltages.

$$V_{pa} = \frac{R_p(-V_{tsc}-V_{tsb}+2V_{tsa})n_pn_s+R_s(-V_{tpc}-V_{tpb}+2V_{tpa})n_p{}^2}{3R_pn_s{}^2+3R_sn_p{}^2} \qquad [7\text{-}1]$$

$$V_{pb} = -\frac{R_p(V_{tsc}-2V_{tsb}+V_{tsa})n_pn_s+R_s(V_{tpc}-2V_{tpb}+V_{tpa})n_p{}^2}{3R_pn_s{}^2+3R_sn_p{}^2} \qquad [7\text{-}2]$$

$$V_{pc} = -\frac{R_p(-2V_{tsc}+V_{tsb}+V_{tsa})n_pn_s+R_s(-2V_{tpc}+V_{tpb}+V_{tpa})n_p{}^2}{3R_pn_s{}^2+3R_sn_p{}^2} \qquad [7\text{-}3]$$

$$I_{pa} = \frac{3R_pV_{tpa}n_s{}^2+R_p(V_{tsc}+V_{tsb}-2V_{tsa})n_pn_s+R_s(V_{tpc}+V_{tpb}+V_{tpa})n_p{}^2}{3R_p{}^2n_s{}^2+3R_pR_sn_p{}^2} \qquad [7\text{-}4]$$

$$I_{pb} = \frac{3R_pV_{tpb}n_s{}^2+R_p(V_{tsc}-2V_{tsb}+V_{tsa})n_pn_s+R_s(V_{tpc}+V_{tpb}+V_{tpa})n_p{}^2}{3R_p{}^2n_s{}^2+3R_pR_sn_p{}^2} \qquad [7\text{-}5]$$

$$I_{pc} = \frac{3R_pV_{tpc}n_s{}^2+R_p(-2V_{tsc}+V_{tsb}+V_{tsa})n_pn_s+R_s(V_{tpc}+V_{tpb}+V_{tpa})n_p{}^2}{3R_p{}^2n_s{}^2+3R_pR_sn_p{}^2} \qquad [7\text{-}6]$$

$$V_{sa} = \frac{R_p(-V_{tsc}-V_{tsb}+2V_{tsa})n_s{}^2+R_s(-V_{tpc}-V_{tpb}+2V_{tpa})n_pn_s}{3R_pn_s{}^2+3R_sn_p{}^2} \qquad [7\text{-}7]$$

$$V_{sb} = -\frac{R_p(V_{tsc}-2V_{tsb}+V_{tsa})n_s{}^2+R_s(V_{tpc}-2V_{tpb}+V_{tpa})n_pn_s}{3R_pn_s{}^2+3R_sn_p{}^2} \qquad [7\text{-}8]$$

$$V_{sc} = -\frac{R_p(-2V_{tsc}+V_{tsb}+V_{tsa})n_s{}^2+R_s(-2V_{tpc}+V_{tpb}+V_{tpa})n_pn_s}{3R_pn_s{}^2+3R_sn_p{}^2} \qquad [7\text{-}9]$$

$$I_{sa} = \frac{R_p(V_{tsc}+V_{tsb}+V_{tsa})n_s{}^2+R_s(V_{tpc}+V_{tpb}-2V_{tpa})n_pn_s+3R_sV_{tsa}n_p{}^2}{3R_pR_sn_s{}^2+3R_s{}^2n_p{}^2} \qquad [7\text{-}10]$$

$$I_{sb} = \frac{R_p(V_{tsc}+V_{tsb}+V_{tsa})n_s{}^2+R_s(V_{tpc}-2V_{tpb}+V_{tpa})n_pn_s+3R_sV_{tsb}n_p{}^2}{3R_pR_sn_s{}^2+3R_s{}^2n_p{}^2} \qquad [7\text{-}11]$$

$$I_{sc} = \frac{R_p(V_{tsc}+V_{tsb}+V_{tsa})n_s{}^2+R_s(-2V_{tpc}+V_{tpb}+V_{tpa})n_pn_s+3R_sV_{tsc}n_p{}^2}{3R_pR_sn_s{}^2+3R_s{}^2n_p{}^2} \qquad [7\text{-}12]$$

$$F_{pa} = \frac{3R_pV_{tpa}n_pn_s{}^2+R_p(V_{tsc}+V_{tsb}-2V_{tsa})n_p{}^2n_s+R_s(V_{tpc}+V_{tpb}+V_{tpa})n_p{}^3}{3R_p{}^2n_s{}^2+3R_pR_sn_p{}^2} \qquad [7\text{-}13]$$

$$F_{pb} = \frac{3R_pV_{tpb}n_pn_s{}^2+R_p(V_{tsc}-2V_{tsb}+V_{tsa})n_p{}^2n_s+R_s(V_{tpc}+V_{tpb}+V_{tpa})n_p{}^3}{3R_p{}^2n_s{}^2+3R_pR_sn_p{}^2} \quad [7\text{-}14]$$

$$F_{pc} = \frac{3R_pV_{tpc}n_pn_s{}^2+R_p(-2V_{tsc}+V_{tsb}+V_{tsa})n_p{}^2n_s+R_s(V_{tpc}+V_{tpb}+V_{tpa})n_p{}^3}{3R_p{}^2n_s{}^2+3R_pR_sn_p{}^2} \quad [7\text{-}15]$$

$$F_{sa} = \frac{R_p(V_{tsc}+V_{tsb}+V_{tsa})n_s{}^3+R_s(V_{tpc}+V_{tpb}-2V_{tpa})n_pn_s{}^2+3R_sV_{tsa}n_p{}^2n_s}{3R_pR_sn_s{}^2+3R_s{}^2n_p{}^2} \quad [7\text{-}16]$$

$$F_{sb} = \frac{R_p(V_{tsc}+V_{tsb}+V_{tsa})n_s{}^3+R_s(V_{tpc}-2V_{tpb}+V_{tpa})n_pn_s{}^2+3R_sV_{tsb}n_p{}^2n_s}{3R_pR_sn_s{}^2+3R_s{}^2n_p{}^2} \quad [7\text{-}17]$$

$$F_{sc} = \frac{R_p(V_{tsc}+V_{tsb}+V_{tsa})n_s{}^3+R_s(-2V_{tpc}+V_{tpb}+V_{tpa})n_pn_s{}^2+3R_sV_{tsc}n_p{}^2n_s}{3R_pR_sn_s{}^2+3R_s{}^2n_p{}^2} \quad [7\text{-}18]$$

$$\Phi_a = \frac{R_p(-V_{tsc}-V_{tsb}+2V_{tsa})n_s+R_s(-V_{tpc}-V_{tpb}+2V_{tpa})n_p}{(3R_pn_s{}^2+3R_sn_p{}^2)\omega\hat{\jmath}} \quad [7\text{-}19]$$

$$\Phi_b = -\frac{R_p(V_{tsc}-2V_{tsb}+V_{tsa})n_s+R_s(V_{tpc}-2V_{tpb}+V_{tpa})n_p}{(3R_pn_s{}^2+3R_sn_p{}^2)\omega\hat{\jmath}} \quad [7\text{-}20]$$

$$\Phi_c = -\frac{R_p(-2V_{tsc}+V_{tsb}+V_{tsa})n_s+R_s(-2V_{tpc}+V_{tpb}+V_{tpa})n_p}{(3R_pn_s{}^2+3R_sn_p{}^2)\omega\hat{\jmath}} \quad [7\text{-}21]$$

$$F_1 = \frac{R_p(V_{tsc}+V_{tsb}+V_{tsa})n_s+R_s(V_{tpc}+V_{tpb}+V_{tpa})n_p}{3R_pR_s} \quad [7\text{-}22]$$

If the primary and secondary voltages are balanced, then:

$$V_{tpa} + V_{tpb} + V_{tpc} = 0 \quad [7\text{-}211]$$

$$V_{tsa} + V_{tsb} + V_{tsc} = 0 \quad [7\text{-}212]$$

If we also set

$$n = \frac{n_p}{n_s} \quad [7\text{-}213]$$

$$R_T = R_p + R_s\, n^2 \quad [2\text{-}107]$$

The variable equations can be simplified to:

$$V_{pa} = \frac{R_p(V_{tsa})n+R_s(V_{tpa})n^2}{R_T} \quad [7\text{-}1a]$$

$$V_{pb} = \frac{R_p(V_{tsb})n+R_s(V_{tpb})n^2}{R_T} \quad [7\text{-}2a]$$

$$V_{pc} = \frac{R_p(V_{tsc})n+R_s(V_{tpc})n^2}{R_T} \quad [7\text{-}3a]$$

$$I_{pa} = \frac{V_{tpa}-V_{tsa}n}{R_T} \quad [7\text{-}4a]$$

$$I_{pb} = \frac{V_{tpb}-V_{tsb}n}{R_T} \quad [7\text{-}5a]$$

$$I_{pc} = \frac{V_{tpc}-V_{tsc}n}{R_T} \quad [7\text{-}6a]$$

$$V_{sa} = \frac{R_p(V_{tsa}) + R_s(V_{tpa})n}{R_T} \quad [7\text{-}7a]$$

$$V_{sb} = \frac{R_p(V_{tsb}) + R_s(V_{tpb})n}{R_T} \quad [7\text{-}8a]$$

$$V_{sc} = \frac{R_p(V_{tsc}) + R_s(V_{tpc})n}{R_T} \quad [7\text{-}9a]$$

$$I_{sa} = \frac{(-V_{tpa})n + V_{tsa}n^2}{R_T} \quad [7\text{-}10a]$$

$$I_{sb} = \frac{(-V_{tpb})n + V_{tsb}n^2}{R_T} \quad [7\text{-}11a]$$

$$I_{sc} = \frac{(-V_{tpc})n + V_{tsc}n^2}{R_T} \quad [7\text{-}12a]$$

$$F_{pa} = \frac{V_{tpa}n_p + (-V_{tsa})n_p n}{R_T} \quad [7\text{-}13a]$$

$$F_{pb} = \frac{V_{tpb}n_p + (-V_{tsb})n_p n}{R_T} \quad [7\text{-}14a]$$

$$F_{pc} = \frac{V_{tpc}n_p + R_p(-V_{tsc})n_p n}{R_T} \quad [7\text{-}15a]$$

$$F_{sa} = \frac{(-V_{tpa})n_p + V_{tsa}n_p n}{R_T} \quad [7\text{-}16a]$$

$$F_{sb} = \frac{(-V_{tpb})n_p + V_{tsb}n_p n}{R_T} \quad [7\text{-}17a]$$

$$F_{sc} = \frac{(-V_{tpc})n_p + V_{tsc}n_p n}{R_T} \quad [7\text{-}18a]$$

$$\Phi_a = \frac{R_p(V_{tsa})\frac{1}{n_s} + R_s(V_{tpa})\frac{n}{n_s}}{R_T\omega\hat{j}} \quad [7\text{-}19a]$$

$$\Phi_b = \frac{R_p(V_{tsb})\frac{1}{n_s} + R_s(V_{tpb})\frac{n}{n_s}}{R_T\omega\hat{j}} \quad [7\text{-}20a]$$

$$\Phi_c = \frac{R_p(V_{tsc})\frac{1}{n_s} + R_s(V_{tpc})\frac{n}{n_s}}{R_T\omega\hat{j}} \quad [7\text{-}21a]$$

$$F_1 = 0 \quad [7\text{-}22a]$$

Conclusions that can be drawn from this analysis include

a) If the primary and secondary voltages are balanced, then the three phases are independent of each other.
b) If the primary and secondary voltages are not balanced, then the three phase currents and voltages are inter-related.
c) With balanced primary and secondary voltages, and if one is only concerned with the voltages and currents at the transformer terminals, then the primary and secondary winding resistance can be combined into R_T as was done in the single-phase transformers,

Figure 20 depicts a three phase transformer with the primary connected in delta and the secondary connected in wye and connected to a balanced wye load (4-wire) with a grounding resistor and a fault on phase A.

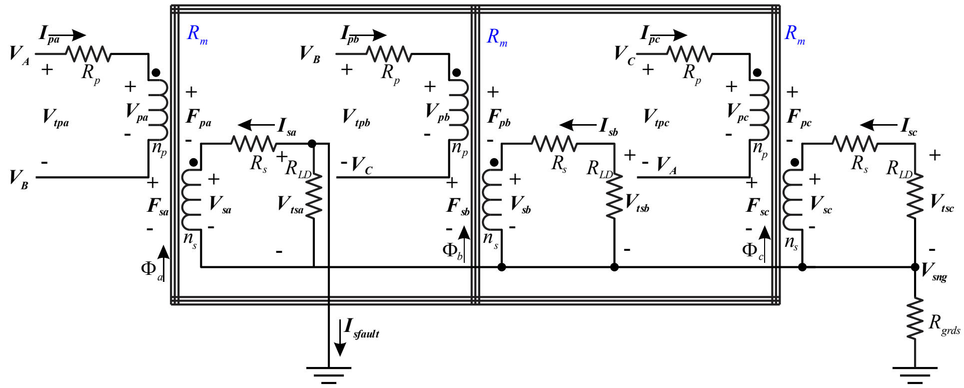

Figure 20: Three-phase transformer connected delta-wye with ground fault

The system of equations can be extended with the following:

The primary voltages:

$$V_{tpa} = V_A - V_B \quad [7\text{-}123]$$

$$V_{tpb} = V_B - V_c \quad [7\text{-}124]$$

$$V_{tpc} = V_C - V_A \quad [7\text{-}125]$$

The secondary currents

$$I_{sa} + \frac{V_{tsa}}{R_{LD}} + I_{sfault} = 0 \quad [7\text{-}126]$$

$$I_{sb} + \frac{V_{tsb}}{R_{LD}} = 0 \quad [7\text{-}127]$$

$$I_{sc} + \frac{V_{tsc}}{R_{LD}} = 0 \quad [7\text{-}128]$$

The grounding resistor

$$V_{sng} + R_{grds} I_{sfault} = 0 \quad [7\text{-}129]$$

$$V_{sng} + V_{tsa} = 0 \quad [7\text{-}130]$$

If R_s and R_p are assumed negligible, the solution to this system of equations is given by:

$$V_{pa} = V_A - V_B \quad [7\text{-}1b]$$

$$V_{pb} = V_B - V_C \quad [7\text{-}2b]$$

$$V_{pc} = V_C - V_A \quad [7\text{-}3b]$$

$$I_{pa} = \frac{(3R_{grds}+2R_{LD})}{3R_{LD}R_{grds}n^2}(V_A - V_B) \quad [7\text{-}4b]$$

$$I_{pb} = \frac{-3R_{grds}V_C + (R_{LD} + 3R_{grds})V_B - R_{LD}V_A}{3R_{LD}R_{grds}n^2} \quad [7\text{-}5b]$$

$$I_{pc} = \frac{3R_{grds}V_C + R_{LD}V_B + (-R_{LD} - 3R_{grds})V_A}{3R_{LD}R_{grds}n^2} \quad [7\text{-}6b]$$

$$V_{sa} = \frac{V_A - V_B}{n} \quad [7\text{-}7b]$$

$$V_{sb} = \frac{V_B - V_C}{n} \quad [7\text{-}8b]$$

$$V_{sc} = \frac{V_C - V_A}{n} \quad [7\text{-}9b]$$

$$I_{sa} = -\frac{R_{grds} + R_{LD}}{R_{LD}R_{grds}n}(V_A - V_B) \quad [7\text{-}10b]$$

$$I_{sb} = -\frac{V_B - V_C}{R_{LD}n} \quad [7\text{-}11b]$$

$$I_{sc} = -\frac{V_C - V_A}{R_{LD}n} \quad [7\text{-}12b]$$

$$F_{pa} = \frac{3R_{grds} + 2R_{LD}}{(3R_{LD}R_{grds})n^2}(V_A - V_B)n_p \quad [7\text{-}13b]$$

$$F_{pb} = \frac{(-3R_{grds}V_C + (R_{LD} + 3R_{grds})V_B - R_{LD}V_A)n_p}{(3R_{LD}R_{grds})n^2} \quad [7\text{-}14b]$$

$$F_{pc} = \frac{(3R_{grds}V_C + R_{LD}V_B + (-R_{LD} - 3R_{grds})V_A)n_p}{(3R_{LD}R_{grds})n^2} \quad [7\text{-}15b]$$

$$F_{sa} = -\frac{(R_{grds} + R_{LD})n_s}{(R_{LD}R_{grds})n}(V_A - V_B) \quad [7\text{-}16b]$$

$$F_{sb} = -\frac{(V_B - V_C)n_s}{R_{LD}n} \quad [7\text{-}17b]$$

$$F_{sc} = -\frac{(V_C - V_A)n_s}{R_{LD}n} \quad [7\text{-}18b]$$

$$\Phi_a = \frac{(V_A - V_B)}{\jmath\omega n_p} \quad [7\text{-}19b]$$

$$\Phi_b = \frac{(V_B - V_C)}{\jmath\omega n_p} \quad [7\text{-}20b]$$

$$\Phi_c = \frac{(V_C - V_A)}{\jmath\omega n_p} \quad [7\text{-}21b]$$

$$F_1 = -\frac{(V_A - V_B)n_s}{(3R_{grds})n} \quad [7\text{-}22b]$$

$$V_{tpa} = V_A - V_B \quad [7\text{-}23b]$$

$$V_{tpb} = V_B - V_C \quad [7\text{-}24b]$$

$$V_{tpc} = V_C - V_A \quad [7\text{-}25b]$$

$$V_{tsa} = \frac{(V_A - V_B)}{n} \quad [7\text{-}26b]$$

$$V_{tsb} = \frac{(V_B - V_C)}{n} \qquad [7\text{-}27b]$$

$$V_{tsc} = \frac{(V_C - V_A)}{n} \qquad [7\text{-}28b]$$

$$V_{sng} = -\frac{(V_A - V_B)}{n} \qquad [7\text{-}29b]$$

$$I_{sfault} = \frac{(V_A - V_B)}{(R_{grds})n} \qquad [7\text{-}30b]$$

Conclusions include:

a) Because F_1 is not zero, the three-phase transformer can be expected to have a different ground fault characteristic than for three single-phase transformers. (The geometry of a single-phase transformer forces F_1 to be identically zero for the three single-phase transformers)
b) The secondary currents are the same as for the case where three single-phase transformers are used.
c) The primary currents are not the same:
Using single-phase transformers, the currents through the primary windings are:

$$I_{tpa} = \frac{(R_{grds} + R_{LD})V_A - (R_{grds} + R_{LD})V_B}{R_{LD}R_{grds}n^2} \qquad [3\text{-}4d]$$

$$I_{tpb} = \frac{V_B - V_C}{R_{LD}n^2} \qquad [3\text{-}5d]$$

$$I_{tpc} = \frac{V_C - V_A}{R_{LD}n^2} \qquad [3\text{-}6d]$$

Using a three-phase transformer, the currents through the primary windings are

$$I_{pa} = \frac{(3R_{grds} + 2R_{LD})}{3R_{LD}R_{grds}n^2}(V_A - V_B) \qquad [7\text{-}4b]$$

$$I_{pb} = \frac{-3R_{grds}V_C + (R_{LD} + 3R_{grds})V_B - R_{LD}V_A}{3R_{LD}R_{grds}n^2} \qquad [7\text{-}5b]$$

$$I_{pc} = \frac{3R_{grds}V_C + R_{LD}V_B + (-R_{LD} - 3R_{grds})V_A}{3R_{LD}R_{grds}n^2} \qquad [7\text{-}6b]$$

d) The primary and secondary voltages are the same for three single-phase transformers and a three-phase transformer.

We now consider a three-phase transformer with two secondary windings. Typically, one of the secondary windings would be configured in delta, and the other in wye. Each would provide power to a rectifier. The rectifiers would either be placed in series or in parallel. The objective is to reduce the harmonic currents in the primary windings.

Figure 21 includes the second set of secondary windings, otherwise known as tertiary windings. If we assume sinusoidal voltages, the following equations are added / modified:

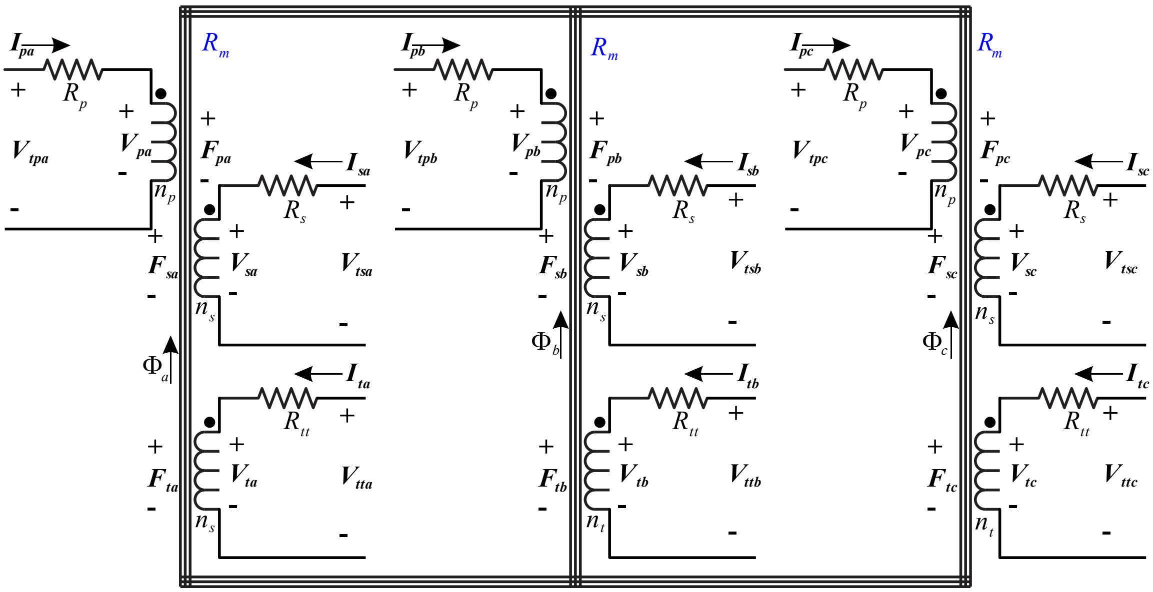

Figure 21: Three-phase transformer with two secondary windings

$$V_{ta} = n_t \omega \hat{J} \Phi_a \quad [7\text{-}131]$$

$$V_{tb} = n_t \omega \hat{J} \Phi_b \quad [7\text{-}132]$$

$$V_{tc} = n_t \omega \hat{J} \Phi_c \quad [7\text{-}133]$$

$$F_{ta} = n_t I_{ta} \quad [7\text{-}134]$$

$$F_{tb} = n_t I_{tb} \quad [7\text{-}135]$$

$$F_{tc} = n_t I_{sc} \quad [7\text{-}136]$$

$$V_{ta} = V_{tta} - R_{tt} I_{ta} \quad [7\text{-}137]$$

$$V_{tb} = V_{ttb} - R_{tt} I_{tb} \quad [7\text{-}138]$$

$$V_{tc} = V_{ttc} - R_{tt} I_{tc} \quad [7\text{-}139]$$

$$F_1 = F_{pa} + F_{sa} + F_{ta} \quad [7\text{-}113a]$$

$$F_1 = F_{pb} + F_{sb} + F_{tb} \quad [7\text{-}114a]$$

$$F_1 = F_{pc} + F_{sc} + F_{tc} \quad [7\text{-}115a]$$

Ignoring, for now, equations 7-123 through 7-130, the system variables can be solved with respect to the terminal voltages:

$$V_{pa} = \frac{n_p(R_p R_s(-V_{ttc} - V_{ttb} + 2V_{tta})n_t + R_{tt}R_p(-V_{tsc} - V_{tsb} + 2V_{tsa})n_s) + R_{tt}R_s(-V_{tpc} - V_{tpb} + 2V_{tpa})n_p{}^2}{3R_pR_s n_t{}^2 + 3R_pR_{tt}n_s{}^2 + 3R_sR_{tt}n_p{}^2} \quad [7\text{-}1c]$$

$$V_{pb} = -\frac{n_p(R_pR_s(V_{ttc}-2V_{ttb}+V_{tta})n_t+R_{tt}R_p(V_{tsc}-2V_{tsb}+V_{tsa})n_s)+R_{tt}R_s(V_{tpc}-2V_{tpb}+V_{tpa})n_p^2}{3R_pR_sn_t^2+3R_pR_{tt}n_s^2+3R_sR_{tt}n_p^2} \qquad \text{[7-2c]}$$

$$V_{pc} = -\frac{n_p(R_pR_s(-2V_{ttc}+V_{ttb}+V_{tta})n_t+R_{tt}R_p(-2V_{tsc}+V_{tsb}+V_{tsa})n_s)+R_{tt}R_s(-2V_{tpc}+V_{tpb}+V_{tpa})n_p^2}{3R_pR_sn_t^2+3R_pR_{tt}n_s^2+3R_sR_{tt}n_p^2} \qquad \text{[7-3c]}$$

$$I_{pa} = \frac{3R_pR_sV_{tpa}n_t^2+n_p(R_pR_s(V_{ttc}+V_{ttb}-2V_{tta})n_t+R_{tt}R_p(V_{tsc}+V_{tsb}-2V_{tsa})n_s)+3R_pR_{tt}V_{tpa}n_s^2+R_{tt}R_s(V_{tpc}+V_{tpb}+V_{tpa})n_p^2}{3R_p\left(R_pR_sn_t^2+R_pR_{tt}n_s^2+R_sR_{tt}n_p^2\right)} \qquad \text{[7-4c]}$$

$$I_{pb} = \frac{3R_pR_sV_{tpb}n_t^2+n_p(R_pR_s(V_{ttc}-2V_{ttb}+V_{tta})n_t+R_{tt}R_p(V_{tsc}-2V_{tsb}+V_{tsa})n_s)+3R_pR_{tt}V_{tpb}n_s^2+R_{tt}R_s(V_{tpc}+V_{tpb}+V_{tpa})n_p^2}{3R_p\left(R_pR_sn_t^2+R_pR_{tt}n_s^2+R_sR_{tt}n_p^2\right)} \qquad \text{[7-5c]}$$

$$I_{pc} = \frac{3R_pR_sV_{tpc}n_t^2+n_p(R_pR_s(-2V_{ttc}+V_{ttb}+V_{tta})n_t+R_{tt}R_p(-2V_{tsc}+V_{tsb}+V_{tsa})n_s)+3R_pR_{tt}V_{tpc}n_s^2+R_{tt}R_s(V_{tpc}+V_{tpb}+V_{tpa})np^2}{3R_p\left(R_pR_sn_t^2+R_pR_{tt}n_s^2+R_sR_{tt}n_p^2\right)} \qquad \text{[7-6c]}$$

$$V_{sa} = \frac{R_pR_s(-V_{ttc}-V_{ttb}+2V_{tta})n_sn_t+R_{tt}R_p(-V_{tsc}-V_{tsb}+2V_{tsa})n_s^2+R_{tt}R_s(-V_{tpc}-V_{tpb}+2V_{tpa})n_pn_s}{3R_pR_sn_t^2+3R_pR_{tt}n_s^2+3R_sR_{tt}n_p^2} \qquad \text{[7-7c]}$$

$$V_{sb} = -\frac{R_pR_s(V_{ttc}-2V_{ttb}+V_{tta})n_sn_t+R_{tt}R_p(V_{tsc}-2V_{tsb}+V_{tsa})n_s^2+R_{tt}R_s(V_{tpc}-2V_{tpb}+V_{tpa})n_pn_s}{3R_pR_sn_t^2+3R_pR_{tt}n_s^2+3R_sR_{tt}n_p^2} \qquad \text{[7-8c]}$$

$$V_{sc} = -\frac{R_pR_s(-2V_{ttc}+V_{ttb}+V_{tta})n_sn_t+R_{tt}R_p(-2V_{tsc}+V_{tsb}+V_{tsa})n_s^2+R_{tt}R_s(-2V_{tpc}+VV_{tpb}+V_{tpa})n_pn_s}{3R_pR_sn_t^2+3R_pR_{tt}n_s^2+3R_sR_{tt}n_p^2} \qquad \text{[7-9c]}$$

$$I_{sa} = \frac{3R_pR_sV_{tsa}n_t^2+R_pR_s(V_{ttc}+V_{ttb}-2V_{tta})n_sn_t+R_{tt}R_p(V_{tsc}+V_{tsb}+V_{tsa})n_s^2+R_{tt}R_s(V_{tpc}+V_{tpb}-2V_{tpa})n_pn_s+3R_{tt}R_sV_{tsa}n_p^2}{3R_s\left(R_pR_sn_t^2+R_pR_{tt}n_s^2+R_sR_{tt}n_p^2\right)} \qquad \text{[7-10c]}$$

$$I_{sb} = \frac{3R_pR_sV_{tsb}n_t^2+R_pR_s(V_{ttc}-2V_{ttb}+V_{tta})n_sn_t+R_{tt}R_p(V_{tsc}+V_{tsb}+V_{tsa})n_s^2+R_{tt}R_s(V_{tpc}-2V_{tpb}+V_{tpa})n_pn_s+3R_{tt}R_sV_{tsb}n_p^2}{3R_s\left(R_pR_sn_t^2+R_pR_{tt}n_s^2+R_sR_{tt}n_p^2\right)} \qquad \text{[7-11c]}$$

$$I_{sc} = \frac{3R_pR_sV_{tsc}n_t^2+R_pR_s(-2V_{ttc}+V_{ttb}+V_{tta})n_sn_t+R_{tt}R_p(V_{tsc}+V_{tsb}+V_{tsa})n_s^2+R_{tt}R_s(-2V_{tpc}+V_{tpb}+V_{tpa})n_pn_s+3R_{tt}R_sV_{tsc}n_p^2}{3R_s\left(R_pR_sn_t^2+R_pR_{tt}n_s^2+R_sR_{tt}n_p^2\right)} \qquad \text{[7-12c]}$$

$$F_{pa} = \frac{3R_pn_p(R_sn_t^2+R_{tt}n_s^2)V_{tpa}+n_p^2(R_pR_s(V_{ttc}+V_{ttb}-2V_{tta})n_t+R_{tt}R_p(V_{tsc}+V_{tsb}-2V_{tsa})n_s)+R_{tt}R_s(V_{tpc}+V_{tpb}+V_{tpa})n_p^3}{3R_p\left(R_pR_sn_t^2+R_pR_{tt}n_s^2+R_sR_{tt}n_p^2\right)} \qquad \text{[7-13c]}$$

$$F_{pb} = \frac{3R_pn_p(R_sn_t^2+R_{tt}n_s^2)V_{tpb}+n_p^2(R_pR_s(V_{ttc}-2V_{ttb}+V_{tta})n_t+R_{tt}R_p(V_{tsc}-2V_{tsb}+V_{tsa})n_s)+R_{tt}R_s(V_{tpc}+V_{tpb}+V_{tpa})n_p^3}{3R_p\left(R_pR_sn_t^2+R_pR_{tt}n_s^2+R_sR_{tt}n_p^2\right)} \qquad \text{[7-14c]}$$

$$F_{pc} = \frac{3R_pn_p(R_sn_t^2+R_{tt}n_s^2)V_{tpc}+n_p^2(R_pR_s(-2V_{ttc}+V_{ttb}+V_{tta})n_t+R_{tt}R_p(-2V_{tsc}+V_{tsb}+V_{tsa})n_s)+R_{tt}R_s(V_{tpc}+V_{tpb}+V_{tpa})n_p^3}{3R_p\left(R_pR_sn_t^2+R_pR_{tt}n_s^2+R_sR_{tt}n_p^2\right)} \qquad \text{[7-15c]}$$

$$F_{sa} = \left(\frac{3R_pR_sV_{tsa}n_t^2+R_pR_s(V_{ttc}+V_{ttb}-2V_{tta})n_sn_t+R_{tt}R_p(V_{tsc}+V_{tsb}+V_{tsa})n_s^2+R_{tt}R_s(V_{tpc}+V_{tpb}-2V_{tpa})n_pn_s+3R_{tt}R_sV_{tsa}n_p^2}{3R_s\left(R_pR_sn_t^2+R_pR_{tt}n_s^2+R_sR_{tt}n_p^2\right)}\right)n_s \qquad \text{[7-16c]}$$

$$F_{sb} = \left(\frac{3R_pR_sV_{tsb}n_t^2+R_pR_s(V_{ttc}-2V_{ttb}+V_{tta})n_sn_t+R_{tt}R_p(V_{tsc}+V_{tsb}+V_{tsa})n_s^2+R_{tt}R_s(V_{tpc}-2V_{tpb}+V_{tpa})n_pn_s+3R_{tt}R_sV_{tsb}n_p^2}{3R_s\left(R_pR_sn_t^2+R_pR_{tt}n_s^2+R_sR_{tt}n_p^2\right)}\right)n_s \qquad \text{[7-17c]}$$

$$F_{sc} = \left(\frac{3R_pR_sV_{tsc}n_t^2+R_pR_s(-2V_{ttc}+V_{ttb}+V_{tta})n_sn_t+R_{tt}R_p(V_{tsc}+V_{tsb}+V_{tsa})n_s^2+R_{tt}R_s(-2V_{tpc}+V_{tpb}+V_{tpa})n_pn_s+3R_{tt}R_sV_{tsc}n_p^2}{3R_s\left(R_pR_sn_t^2+R_pR_{tt}n_s^2+R_sR_{tt}n_p^2\right)}\right)n_s \qquad \text{[7-18c]}$$

$$\Phi_a = \frac{R_pR_s(-V_{ttc}-V_{ttb}+2V_{tta})n_t+R_pR_{tt}(-V_{tsc}-V_{tsb}+2V_{tsa})n_s+R_{tt}R_s(-V_{tpc}-V_{tpb}+2V_{tpa})n_p}{3(R_pR_sn_t^2+R_pR_{tt}n_s^2+R_sR_{tt}n_p^2)\omega\hat{\jmath}} \qquad \text{[7-19c]}$$

$$\Phi_b = -\frac{R_pR_s(V_{ttc}-2V_{ttb}+V_{tta})n_t+R_pR_{tt}(V_{tsc}-2V_{tsb}+V_{tsa})n_s+R_{tt}R_s(V_{tpc}-2V_{tpb}+V_{tpa})n_p}{3(R_pR_sn_t^2+R_pR_{tt}n_s^2+R_sR_{tt}n_p^2)\omega\hat{\jmath}} \qquad \text{[7-20c]}$$

$$\Phi_c = -\frac{R_pR_s(-2V_{ttc}+V_{ttb}+V_{tta})n_t+R_pR_{tt}(-2V_{tsc}+V_{tsb}+V_{tsa})n_s+R_{tt}R_s(-2V_{tpc}+V_{tpb}+V_{tpa})n_p}{3(R_pR_sn_t^2+R_pR_{tt}n_s^2+R_sR_{tt}n_p^2)\omega\hat{\jmath}} \qquad \text{[7-21c]}$$

$$F_1 = \frac{R_pR_s(V_{ttc}+V_{ttb}+V_{tta})n_t+R_pR_{tt}(V_{tsc}+V_{tsb}+V_{tsa})n_s+R_{tt}R_s(V_{tpc}+V_{tpb}+V_{tpa})n_p}{3R_pR_sR_{tt}} \qquad \text{[7-22c]}$$

$$V_{ta} = \frac{R_pR_s(-V_{ttc}-V_{ttb}+2V_{tta})n_t{}^2+R_{tt}R_p(-V_{tsc}-V_{tsb}+2V_{tsa})n_tn_s+R_{tt}R_s(-V_{tpc}-V_{tpb}+2V_{tpa})n_tn_p}{3R_pR_sn_t{}^2+3R_pR_{tt}n_s{}^2+3R_sR_{tt}n_p{}^2} \quad [7\text{-}31c]$$

$$V_{tb} = -\frac{R_pR_s(V_{ttc}-2V_{ttb}+V_{tta})n_t{}^2+R_{tt}R_p(V_{tsc}-2V_{tsb}+V_{tsa})n_tn_s+R_{tt}R_s(V_{tpc}-2V_{tpb}+V_{tpa})n_tn_p}{3R_pR_sn_t{}^2+3R_pR_{tt}n_s{}^2+3R_sR_{tt}n_p{}^2} \quad [7\text{-}32c]$$

$$V_{tc} = -\frac{R_pR_s(-2V_{ttc}+V_{ttb}+V_{tta})n_t{}^2+R_{tt}R_p(-2V_{tsc}+V_{tsb}+V_{tsa})n_tn_s+R_{tt}R_s(-2V_{tpc}+V_{tpb}+V_{tpa})n_tn_p}{3R_pR_sn_t{}^2+3R_pR_{tt}n_s{}^2+3R_sR_{tt}n_p{}^2} \quad [7\text{-}33c]$$

$$I_{ta} = \frac{R_pR_s(V_{ttc}+V_{ttb}+V_{tta})n_t{}^2+R_{tt}R_p(V_{tsc}+V_{tsb}-2V_{tsa})n_tn_s+R_{tt}R_s(V_{tpc}+V_{tpb}-2V_{tpa})n_tn_p+3R_{tt}(R_pn_s{}^2+3R_sn_p{}^2)V_{tta}}{3R_{tt}(R_pR_sn_t{}^2+R_pR_{tt}n_s{}^2+R_sR_{tt}n_p{}^2)} \quad [7\text{-}34c]$$

$$I_{tb} = \frac{R_pR_s(V_{ttc}+V_{ttb}+V_{tta})n_t{}^2+R_{tt}R_p(V_{tsc}-2V_{tsb}+V_{tsa})n_tn_s+R_{tt}R_s(V_{tpc}-2V_{tpb}+V_{tpa})n_tn_p+3R_{tt}(R_pn_s{}^2+3R_sn_p{}^2)V_{ttb}}{3R_{tt}(R_pR_sn_t{}^2+R_pR_{tt}n_s{}^2+R_sR_{tt}n_p{}^2)} \quad [7\text{-}35c]$$

$$I_{tc} = \frac{R_pR_s(V_{ttc}+V_{ttb}+V_{tta})n_t{}^2+R_{tt}R_p(-2V_{tsc}+V_{tsb}+V_{tsa})n_tn_s+R_{tt}R_s(-2V_{tpc}+V_{tpb}+V_{tpa})n_tn_p+3R_{tt}(R_pn_s{}^2+3R_sn_p{}^2)V_{ttc}}{3R_{tt}(R_pR_sn_t{}^2+R_pR_{tt}n_s{}^2+R_sR_{tt}n_p{}^2)} \quad [7\text{-}36c]$$

$$F_{ta} = \left(\frac{R_pR_s(V_{ttc}+V_{ttb}+V_{tta})n_t{}^2+R_{tt}R_p(V_{tsc}+V_{tsb}-2V_{tsa})n_tn_s+R_{tt}R_s(V_{tpc}+V_{tpb}-2V_{tpa})n_tn_p+3R_{tt}(R_pn_s{}^2+3R_sn_p{}^2)V_{tta}}{3R_{tt}(R_pR_sn_t{}^2+R_pR_{tt}n_s{}^2+R_sR_{tt}n_p{}^2)}\right)n_t \quad [7\text{-}37c]$$

$$F_{tb} = \left(\frac{R_pR_s(V_{ttc}+V_{ttb}+V_{tta})n_t{}^2+R_{tt}R_p(V_{tsc}-2V_{tsb}+V_{tsa})n_tn_s+R_{tt}R_s(V_{tpc}-2V_{tpb}+V_{tpa})n_tn_p+3R_{tt}(R_pn_s{}^2+3R_sn_p{}^2)V_{ttb}}{3R_{tt}(R_pR_sn_t{}^2+R_pR_{tt}n_s{}^2+R_sR_{tt}n_p{}^2)}\right)n_t \quad [7\text{-}38c]$$

$$F_{tc} = \left(\frac{R_pR_s(V_{ttc}+V_{ttb}+V_{tta})n_t{}^2+R_{tt}R_p(-2V_{tsc}+V_{tsb}+V_{tsa})n_tn_s+R_{tt}R_s(-2V_{tpc}+V_{tpb}+V_{tpa})n_tn_p+3R_{tt}(R_pn_s{}^2+3R_sn_p{}^2)V_{ttc}}{3R_{tt}(R_pR_sn_t{}^2+R_pR_{tt}n_s{}^2+R_sR_{tt}n_p{}^2)}\right)n_t \quad [7\text{-}39c]$$

If the voltages are all balanced, then the expressions above can be simplified.

$$V_{pa} = \frac{R_pR_sV_{tta}n_tn_p+R_{tt}R_pV_{tsa}n_sn_p+R_{tt}R_sV_{tpa}n_p{}^2}{R_pR_sn_t{}^2+R_pR_{tt}n_s{}^2+R_sR_{tt}n_p{}^2} \quad [7\text{-}1d]$$

$$V_{pb} = \frac{R_pR_sV_{ttb}n_tn_p+R_{tt}R_pV_{tsb}n_sn_p+R_{tt}R_sV_{tpb}n_p{}^2}{R_pR_sn_t{}^2+R_pR_{tt}n_s{}^2+R_sR_{tt}n_p{}^2} \quad [7\text{-}2d]$$

$$V_{pc} = \frac{R_pR_sV_{ttc}n_tn_p+R_{tt}R_pV_{tsc}n_sn_p+R_{tt}R_sV_{tpc}n_p{}^2}{R_pR_sn_t{}^2+R_pR_{tt}n_s{}^2+R_sR_{tt}n_p{}^2} \quad [7\text{-}3d]$$

$$I_{pa} = \frac{-R_sV_{tta}n_tn_p-R_{tt}V_{tsa}n_sn_p+V_{tpa}(R_{tt}n_s{}^2+R_sn_t{}^2)}{R_pR_sn_t{}^2+R_pR_{tt}n_s{}^2+R_sR_{tt}n_p{}^2} \quad [7\text{-}4d]$$

$$I_{pb} = \frac{-R_sV_{ttb}n_tn_p-R_{tt}V_{tsb}n_sn_p+V_{tpb}(R_{tt}n_s{}^2+R_sn_t{}^2)}{R_pR_sn_t{}^2+R_pR_{tt}n_s{}^2+R_sR_{tt}n_p{}^2} \quad [7\text{-}5d]$$

$$I_{pc} = \frac{-R_sV_{ttc}n_tn_p-R_{tt}V_{tsc}n_sn_p+V_{tpc}(R_{tt}n_s{}^2+R_sn_t{}^2)}{R_pR_sn_t{}^2+R_pR_{tt}n_s{}^2+R_sR_{tt}n_p{}^2} \quad [7\text{-}6d]$$

$$V_{sa} = \frac{R_pR_sV_{tta}n_sn_t+R_{tt}R_pV_{tsa}n_s{}^2+R_{tt}R_sV_{tpa}n_pn_s}{R_pR_sn_t{}^2+R_pR_{tt}n_s{}^2+R_sR_{tt}n_p{}^2} \quad [7\text{-}7d]$$

$$V_{sb} = \frac{R_pR_sV_{ttb}n_sn_t+R_{tt}R_pV_{tsb}n_s{}^2+R_{tt}R_sV_{tpb}n_pn_s}{R_pR_sn_t{}^2+R_pR_{tt}n_s{}^2+R_sR_{tt}n_p{}^2} \quad [7\text{-}8d]$$

$$V_{sc} = \frac{R_pR_sV_{ttc}n_sn_t+R_{tt}R_pV_{tsc}n_s{}^2+R_{tt}R_sV_{tpc}n_pn_s}{R_pR_sn_t{}^2+R_pR_{tt}n_s{}^2+R_sR_{tt}n_p{}^2} \quad [7\text{-}9d]$$

$$I_{sa} = \frac{-R_pV_{tta}n_sn_t-R_{tt}V_{tpa}n_pn_s+V_{tsa}(R_{tt}n_p{}^2+R_pn_t{}^2)}{R_pR_sn_t{}^2+R_pR_{tt}n_s{}^2+R_sR_{tt}n_p{}^2} \quad [7\text{-}10d]$$

$$I_{sb} = \frac{-R_pV_{ttb}n_sn_t - R_{tt}V_{tpb}n_pn_s + V_{tsb}\left(R_{tt}{n_p}^2 + R_p{n_t}^2\right)}{R_pR_s{n_t}^2 + R_pR_{tt}{n_s}^2 + R_sR_{tt}{n_p}^2} \qquad [7\text{-}11d]$$

$$I_{sc} = \frac{-R_pV_{ttc}n_sn_t - R_{tt}V_{tpc}n_pn_s + V_{tsc}\left(R_{tt}{n_p}^2 + R_p{n_t}^2\right)}{R_pR_s{n_t}^2 + R_pR_{tt}{n_s}^2 + R_sR_{tt}{n_p}^2} \qquad [7\text{-}12d]$$

$$F_{pa} = \frac{n_p\left(R_s{n_t}^2 + R_{tt}{n_s}^2\right)V_{tpa} - {n_p}^2\left(R_sV_{tta}n_t + R_{tt}V_{tsa}n_s\right)}{R_pR_s{n_t}^2 + R_pR_{tt}{n_s}^2 + R_sR_{tt}{n_p}^2} \qquad [7\text{-}13d]$$

$$F_{pb} = \frac{n_p\left(R_s{n_t}^2 + R_{tt}{n_s}^2\right)V_{tpb} - {n_p}^2\left(R_sV_{ttb}n_t + R_{tt}V_{tsb}n_s\right)}{R_pR_s{n_t}^2 + R_pR_{tt}{n_s}^2 + R_sR_{tt}{n_p}^2} \qquad [7\text{-}14d]$$

$$F_{pc} = \frac{n_p\left(R_s{n_t}^2 + R_{tt}{n_s}^2\right)V_{tpc} - {n_p}^2\left(R_sV_{ttc}n_t + R_{tt}V_{tsc}n_s\right)}{R_pR_s{n_t}^2 + R_pR_{tt}{n_s}^2 + R_sR_{tt}{n_p}^2} \qquad [7\text{-}15d]$$

$$F_{sa} = \left(\frac{-R_pV_{tta}n_sn_t - R_{tt}V_{tpa}n_pn_s + V_{tsa}\left(R_{tt}{n_p}^2 + R_p{n_t}^2\right)}{R_pR_s{n_t}^2 + R_pR_{tt}{n_s}^2 + R_sR_{tt}{n_p}^2}\right)n_s \qquad [7\text{-}16d]$$

$$F_{sb} = \left(\frac{-R_pV_{ttb}n_sn_t - R_{tt}V_{tpb}n_pn_s + V_{tsb}\left(R_{tt}{n_p}^2 + R_p{n_t}^2\right)}{R_pR_s{n_t}^2 + R_pR_{tt}{n_s}^2 + R_sR_{tt}{n_p}^2}\right)n_s \qquad [7\text{-}17d]$$

$$F_{sc} = \left(\frac{-R_pV_{ttc}n_sn_t - R_{tt}V_{tpc}n_pn_s + V_{tsc}\left(R_{tt}{n_p}^2 + R_p{n_t}^2\right)}{R_pR_s{n_t}^2 + R_pR_{tt}{n_s}^2 + R_sR_{tt}{n_p}^2}\right)n_s \qquad [7\text{-}18d]$$

$$\Phi_a = \frac{R_pR_sV_{tta}n_t + R_pR_{tt}V_{tsa}n_s + R_{tt}R_sV_{tpa}n_p}{\left(R_pR_s{n_t}^2 + R_pR_{tt}{n_s}^2 + R_sR_{tt}{n_p}^2\right)\omega\hat{\jmath}} \qquad [7\text{-}19d]$$

$$\Phi_b = \frac{R_pR_sV_{ttb}n_t + R_pR_{tt}V_{tsb}n_s + R_{tt}R_sV_{tpb}n_p}{\left(R_pR_s{n_t}^2 + R_pR_{tt}{n_s}^2 + R_sR_{tt}{n_p}^2\right)\omega\hat{\jmath}} \qquad [7\text{-}20d]$$

$$\Phi_c = \frac{R_pR_sV_{ttc}n_t + R_pR_{tt}V_{tsc}n_s + R_{tt}R_sV_{tpc}n_p}{\left(R_pR_s{n_t}^2 + R_pR_{tt}{n_s}^2 + R_sR_{tt}{n_p}^2\right)\omega\hat{\jmath}} \qquad [7\text{-}21d]$$

$$F_1 = 0 \qquad [7\text{-}22d]$$

$$V_{ta} = \frac{R_pR_sV_{tta}{n_t}^2 + R_{tt}R_pV_{tsa}n_tn_s + R_{tt}R_sV_{tpa}n_tn_p}{R_pR_s{n_t}^2 + R_pR_{tt}{n_s}^2 + R_sR_{tt}{n_p}^2} \qquad [7\text{-}31d]$$

$$V_{tb} = \frac{R_pR_sV_{ttb}{n_t}^2 + R_{tt}R_pV_{tsb}n_tn_s + R_{tt}R_sV_{tpb}n_tn_p}{R_pR_s{n_t}^2 + R_pR_{tt}{n_s}^2 + R_sR_{tt}{n_p}^2} \qquad [7\text{-}32d]$$

$$V_{tc} = \frac{R_pR_sV_{ttc}{n_t}^2 + R_{tt}R_pV_{tsc}n_tn_s + R_{tt}R_sV_{tpc}n_tn_p}{R_pR_s{n_t}^2 + R_pR_{tt}{n_s}^2 + R_sR_{tt}{n_p}^2} \qquad [7\text{-}33d]$$

$$I_{ta} = \frac{-R_pV_{tsa}n_tn_s - R_sV_{tpa}n_tn_p + \left(R_p{n_s}^2 + 3R_s{n_p}^2\right)V_{tta}}{\left(R_pR_s{n_t}^2 + R_pR_{tt}{n_s}^2 + R_sR_{tt}{n_p}^2\right)} \qquad [7\text{-}34d]$$

$$I_{tb} = \frac{-R_pV_{tsb}n_tn_s - R_sV_{tpb}n_tn_p + \left(R_p{n_s}^2 + 3R_s{n_p}^2\right)V_{ttb}}{\left(R_pR_s{n_t}^2 + R_pR_{tt}{n_s}^2 + R_sR_{tt}{n_p}^2\right)} \qquad [7\text{-}35d]$$

$$I_{tc} = \frac{-R_pV_{tsc}n_tn_s - R_sV_{tpc}n_tn_p + \left(R_p{n_s}^2 + 3R_s{n_p}^2\right)V_{ttc}}{\left(R_pR_s{n_t}^2 + R_pR_{tt}{n_s}^2 + R_sR_{tt}{n_p}^2\right)} \qquad [7\text{-}36d]$$

$$F_{ta} = \left(\frac{-R_pV_{tsa}n_tn_s - R_sV_{tpa}n_tn_p + \left(R_p{n_s}^2 + 3R_s{n_p}^2\right)V_{tta}}{\left(R_pR_s{n_t}^2 + R_pR_{tt}{n_s}^2 + R_sR_{tt}{n_p}^2\right)}\right)n_t \qquad [7\text{-}37d]$$

$$F_{tb} = \left(\frac{-R_pV_{tsb}n_tn_s - R_sV_{tpb}n_tn_p + \left(R_p{n_s}^2 + 3R_s{n_p}^2\right)V_{ttb}}{\left(R_pR_s{n_t}^2 + R_pR_{tt}{n_s}^2 + R_sR_{tt}{n_p}^2\right)}\right)n_t \qquad [7\text{-}38d]$$

$$F_{tc} = \left(\frac{-R_p V_{tsc} n_t n_s - R_s V_{tpc} n_t n_p + (R_p n_s{}^2 + 3R_s n_p{}^2) V_{ttc}}{(R_p R_s n_t{}^2 + R_p R_{tt} n_s{}^2 + R_s R_{tt} n_p{}^2)}\right) n_t \qquad [7\text{-}39d]$$

Figure 22 configures the primary as a delta winding, the secondary winding as a wye winding with a wye resistive load, and the tertiary winding as a delta winding with a wye resistive load.

Equations 7-123 through 7-130 are added back into the system of equations with the exception that the following equations are modified:

$$I_{sa} + \frac{V_{tsa}}{R_{LD}} = 0 \qquad [7\text{-}126e]$$

$$I_{sfault} = 0 \qquad [7\text{-}130e]$$

The following equations are added to implement the delta winding and load on the tertiary winding:

$$I_{ta} + I_{tlda} - I_{tc} = 0 \qquad [7\text{-}140]$$

$$I_{tb} + I_{tldb} - I_{ta} = 0 \qquad [7\text{-}141]$$

$$I_{tc} + I_{tldc} - I_{tb} = 0 \qquad [7\text{-}142]$$

$$I_{tlda} + I_{tldb} + I_{tldc} + I_{tfault} = 0 \qquad [7\text{-}143]$$

$$V_{tng} + R_{grdt} I_{tfault} = 0 \qquad [7\text{-}144]$$

$$V_{tta} + V_{ttb} + V_{ttc} = 0 \qquad [7\text{-}145]$$

$$-I_{tlda} R_{tLD} + V_{tta} + I_{tldb} R_{tLD} = 0 \qquad [7\text{-}146]$$

$$-I_{tldb} R_{tLD} + V_{ttb} + I_{tldc} R_{tLD} = 0 \qquad [7\text{-}147]$$

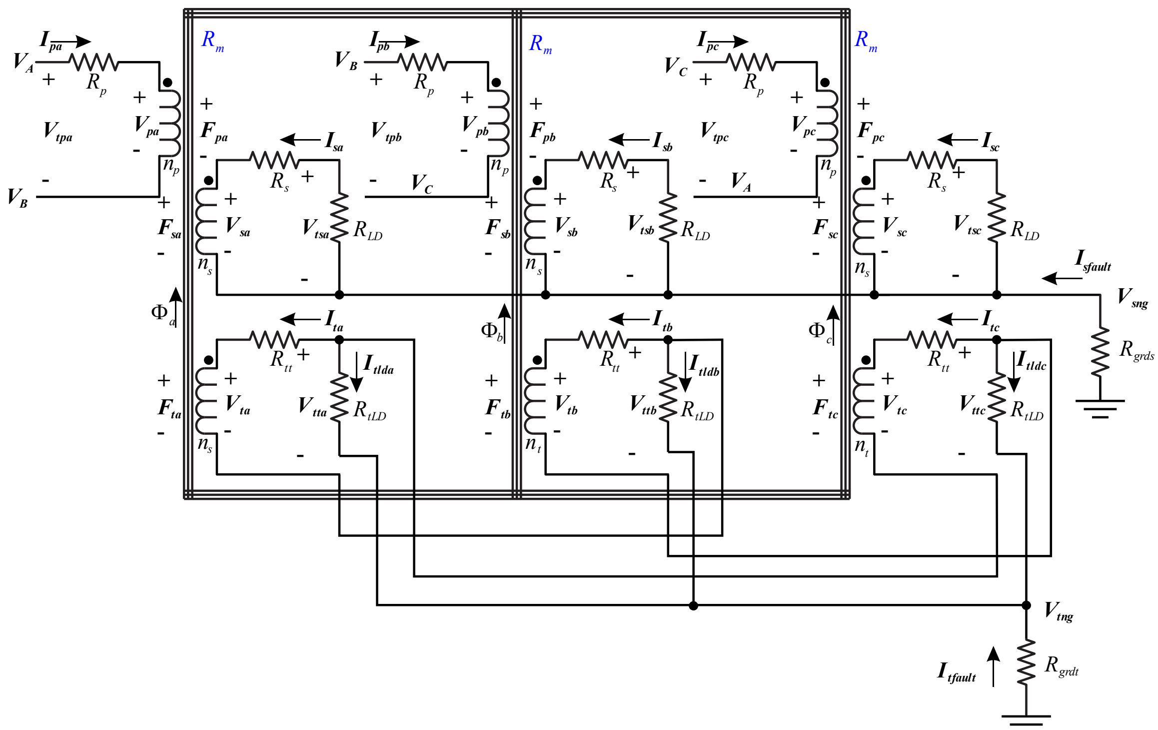

Figure 22: Three-phase transformer with two secondary windings – with resistive loads

With balanced voltages on the primary windings, The solution to this set of equations is given by:

$$V_{pa} = \frac{(R_{tt}R_s+3R_{tLD}R_s+R_{tt}R_{LD}+3R_{LD}R_{tLD})(V_A-V_B){n_p}^2}{R_p{n_t}^2(R_s+R_{LD})+R_p(R_{tt}+3R_{tLD}){n_s}^2+(R_s(R_{tt}+3R_{tLD})+R_{LD}R_{tt}+3R_{LD}R_{tLD}){n_p}^2} \quad [7\text{-}1f]$$

$$V_{pb} = \frac{(R_{tt}R_s+3R_{tLD}R_s+R_{tt}R_{LD}+3R_{LD}R_{tLD})(V_B-V_C){n_p}^2}{R_p{n_t}^2(R_s+R_{LD})+R_p(R_{tt}+3R_{tLD}){n_s}^2+(R_s(R_{tt}+3R_{tLD})+R_{LD}R_{tt}+3R_{LD}R_{tLD}){n_p}^2} \quad [7\text{-}2f]$$

$$V_{pc} = \frac{(R_{tt}R_s+3R_{tLD}R_s+R_{tt}R_{LD}+3R_{LD}R_{tLD})(V_C-V_A){n_p}^2}{R_p{n_t}^2(R_s+R_{LD})+R_p(R_{tt}+3R_{tLD}){n_s}^2+(R_s(R_{tt}+3R_{tLD})+R_{LD}R_{tt}+3R_{LD}R_{tLD}){n_p}^2} \quad [7\text{-}3f]$$

$$I_{pa} = \frac{\left((R_{LD}+R_s){n_t}^2+(R_{tt}+3R_{tLD}){n_s}^2\right)(V_A-V_B)}{R_p{n_t}^2(R_s+R_{LD})+R_p(R_{tt}+3R_{tLD}){n_s}^2+(R_s(R_{tt}+3R_{tLD})+R_{LD}R_{tt}+3R_{LD}R_{tLD}){n_p}^2} \quad [7\text{-}4f]$$

$$I_{pb} = \frac{\left((R_{LD}+R_s){n_t}^2+(R_{tt}+3R_{tLD}){n_s}^2\right)(V_B-V_C)}{R_p{n_t}^2(R_s+R_{LD})+R_p(R_{tt}+3R_{tLD}){n_s}^2+(R_s(R_{tt}+3R_{tLD})+R_{LD}R_{tt}+3R_{LD}R_{tLD}){n_p}^2} \quad [7\text{-}5f]$$

$$I_{pc} = \frac{\left((R_{LD}+R_s){n_t}^2+(R_{tt}+3R_{tLD}){n_s}^2\right)(V_C-V_A)}{R_p{n_t}^2(R_s+R_{LD})+R_p(R_{tt}+3R_{tLD}){n_s}^2+(R_s(R_{tt}+3R_{tLD})+R_{LD}R_{tt}+3R_{LD}R_{tLD}){n_p}^2} \quad [7\text{-}6f]$$

$$V_{sa} = \frac{(R_{tt}R_s+3R_{tLD}R_s+R_{tt}R_{LD}+3R_{LD}R_{tLD})(V_A-V_B)n_p n_s}{R_p{n_t}^2(R_s+R_{LD})+R_p(R_{tt}+3R_{tLD}){n_s}^2+(R_s(R_{tt}+3R_{tLD})+R_{LD}R_{tt}+3R_{LD}R_{tLD}){n_p}^2} \quad [7\text{-}7f]$$

$$V_{sb} = \frac{(R_{tt}R_s+3R_{tLD}R_s+R_{tt}R_{LD}+3R_{LD}R_{tLD})(V_B-V_C)n_p n_s}{R_p{n_t}^2(R_s+R_{LD})+R_p(R_{tt}+3R_{tLD}){n_s}^2+(R_s(R_{tt}+3R_{tLD})+R_{LD}R_{tt}+3R_{LD}R_{tLD}){n_p}^2} \quad [7\text{-}8f]$$

$$V_{sc} = \frac{(R_{tt}R_s+3R_{tLD}R_s+R_{tt}R_{LD}+3R_{LD}R_{tLD})(V_C-V_A)n_p n_s}{R_p{n_t}^2(R_s+R_{LD})+R_p(R_{tt}+3R_{tLD}){n_s}^2+(R_s(R_{tt}+3R_{tLD})+R_{LD}R_{tt}+3R_{LD}R_{tLD}){n_p}^2} \quad [7\text{-}9f]$$

$$I_{sa} = -\frac{(R_{tt}+3R_{tLD})(V_A-V_B)n_p n_s}{R_p n_t^2(R_s+R_{LD})+R_p(R_{tt}+3R_{tLD})n_s^2+(R_s(R_{tt}+3R_{tLD})+R_{LD}R_{tt}+3R_{LD}R_{tLD})n_p^2} \quad [7\text{-}10f]$$

$$I_{sb} = -\frac{(R_{tt}+3R_{tLD})(V_B-V_C)n_p n_s}{R_p n_t^2(R_s+R_{LD})+R_p(R_{tt}+3R_{tLD})n_s^2+(R_s(R_{tt}+3R_{tLD})+R_{LD}R_{tt}+3R_{LD}R_{tLD})n_p^2} \quad [7\text{-}11f]$$

$$I_{sc} = -\frac{(R_{tt}+3R_{tLD})(V_C-V_A)n_p n_s}{R_p n_t^2(R_s+R_{LD})+R_p(R_{tt}+3R_{tLD})n_s^2+(R_s(R_{tt}+3R_{tLD})+R_{LD}R_{tt}+3R_{LD}R_{tLD})n_p^2} \quad [7\text{-}12f]$$

$$F_1 = 0 \quad [7\text{-}22f]$$

$$V_{tpa} = V_A - V_{\boldsymbol{B}} \quad [7\text{-}23f]$$

$$V_{tpb} = V_B - V_C \quad [7\text{-}24f]$$

$$V_{tpc} = V_C - V_A \quad [7\text{-}25f]$$

$$V_{tsa} = \frac{R_{LD}(R_{tt}+3R_{tLD})(V_A-V_B)n_p n_s}{R_p n_t^2(R_s+R_{LD})+R_p(R_{tt}+3R_{tLD})n_s^2+(R_s(R_{tt}+3R_{tLD})+R_{LD}R_{tt}+3R_{LD}R_{tLD})n_p^2} \quad [7\text{-}26f]$$

$$V_{tsb} = \frac{R_{LD}(R_{tt}+3R_{tLD})(V_B-V_C)n_p n_s}{R_p n_t^2(R_s+R_{LD})+R_p(R_{tt}+3R_{tLD})n_s^2+(R_s(R_{tt}+3R_{tLD})+R_{LD}R_{tt}+3R_{LD}R_{tLD})n_p^2} \quad [7\text{-}27f]$$

$$V_{tsc} = \frac{R_{LD}(R_{tt}+3R_{tLD})(V_C-V_A)n_p n_s}{R_p n_t^2(R_s+R_{LD})+R_p(R_{tt}+3R_{tLD})n_s^2+(R_s(R_{tt}+3R_{tLD})+R_{LD}R_{tt}+3R_{LD}R_{tLD})n_p^2} \quad [7\text{-}28f]$$

$$V_{ta} = \frac{(R_{tt}R_{LD}+3R_{LD}R_{tLD}+R_{tt}R_s+3R_sR_{tLD})(V_A-V_B)n_p n_t}{R_p n_t^2(R_s+R_{LD})+R_p(R_{tt}+3R_{tLD})n_s^2+(R_s(R_{tt}+3R_{tLD})+R_{LD}R_{tt}+3R_{LD}R_{tLD})n_p^2} \quad [7\text{-}31f]$$

$$V_{tb} = \frac{(R_{tt}R_{LD}+3R_{LD}R_{tLD}+R_{tt}R_s+3R_sR_{tLD})(V_B-V_C)n_p n_t}{R_p n_t^2(R_s+R_{LD})+R_p(R_{tt}+3R_{tLD})n_s^2+(R_s(R_{tt}+3R_{tLD})+R_{LD}R_{tt}+3R_{LD}R_{tLD})n_p^2} \quad [7\text{-}32f]$$

$$V_{tc} = \frac{(R_{tt}R_{LD}+3R_{LD}R_{tLD}+R_{tt}R_s+3R_sR_{tLD})(V_C-V_A)n_p n_t}{R_p n_t^2(R_s+R_{LD})+R_p(R_{tt}+3R_{tLD})n_s^2+(R_s(R_{tt}+3R_{tLD})+R_{LD}R_{tt}+3R_{LD}R_{tLD})n_p^2} \quad [7\text{-}33f]$$

$$I_{ta} = -\frac{(R_{LD}+R_s)(V_A-V_B)n_p n_t}{R_p n_t^2(R_s+R_{LD})+R_p(R_{tt}+3R_{tLD})n_s^2+(R_s(R_{tt}+3R_{tLD})+R_{LD}R_{tt}+3R_{LD}R_{tLD})n_p^2} \quad [7\text{-}34f]$$

$$I_{tb} = -\frac{(R_{LD}+R_s)(V_B-V_C)n_p n_t}{R_p n_t^2(R_s+R_{LD})+R_p(R_{tt}+3R_{tLD})n_s^2+(R_s(R_{tt}+3R_{tLD})+R_{LD}R_{tt}+3R_{LD}R_{tLD})n_p^2} \quad [7\text{-}35f]$$

$$I_{tc} = -\frac{(R_{LD}+R_s)(V_C-V_A)n_p n_t}{R_p n_t^2(R_s+R_{LD})+R_p(R_{tt}+3R_{tLD})n_s^2+(R_s(R_{tt}+3R_{tLD})+R_{LD}R_{tt}+3R_{LD}R_{tLD})n_p^2} \quad [7\text{-}36f]$$

$$V_{tta} = \frac{3R_{tLD}(R_{LD}+R_s)(V_A-V_B)n_p n_t}{R_p n_t^2(R_s+R_{LD})+R_p(R_{tt}+3R_{tLD})n_s^2+(R_s(R_{tt}+3R_{tLD})+R_{LD}R_{tt}+3R_{LD}R_{tLD})n_p^2} \quad [7\text{-}40f]$$

$$V_{ttb} = \frac{3R_{tLD}(R_{LD}+R_s)(V_B-V_C)n_p n_t}{R_p n_t^2(R_s+R_{LD})+R_p(R_{tt}+3R_{tLD})n_s^2+(R_s(R_{tt}+3R_{tLD})+R_{LD}R_{tt}+3R_{LD}R_{tLD})n_p^2} \quad [7\text{-}41f]$$

$$V_{ttc} = \frac{3R_{tLD}(R_{LD}+R_s)(V_C-V_A)n_p n_t}{R_p n_t^2(R_s+R_{LD})+R_p(R_{tt}+3R_{tLD})n_s^2+(R_s(R_{tt}+3R_{tLD})+R_{LD}R_{tt}+3R_{LD}R_{tLD})n_p^2} \quad [7\text{-}42f]$$

$$I_{tlda} = \frac{3(R_{LD}+R_s)V_A n_p n_t}{R_p n_t^2(R_s+R_{LD})+R_p(R_{tt}+3R_{tLD})n_s^2+(R_s(R_{tt}+3R_{tLD})+R_{LD}R_{tt}+3R_{LD}R_{tLD})n_p^2} \quad [7\text{-}43f]$$

$$I_{tldb} = \frac{3(R_{LD}+R_s)V_B n_p n_t}{R_p n_t^2(R_s+R_{LD})+R_p(R_{tt}+3R_{tLD})n_s^2+(R_s(R_{tt}+3R_{tLD})+R_{LD}R_{tt}+3R_{LD}R_{tLD})n_p^2} \quad [7\text{-}44f]$$

$$I_{tldc} = \frac{3(R_{LD}+R_s)V_C n_p n_t}{R_p n_t^2(R_s+R_{LD})+R_p(R_{tt}+3R_{tLD})n_s^2+(R_s(R_{tt}+3R_{tLD})+R_{LD}R_{tt}+3R_{LD}R_{tLD})n_p^2} \quad [7\text{-}45f]$$

If we assume R_p, R_s, and R_{tt} are negligible, then the results can be simplified

$$V_{pa} = V_A - V_B \quad [7\text{-}1g]$$

$$V_{pb} = V_B - V_C \quad [7\text{-}2g]$$

$$V_{pc} = V_C - V_A \quad [7\text{-}3g]$$

$$I_{pa} = \left(\frac{R_{LD}n_t^2+3R_{tLD}n_s^2}{3R_{LD}R_{tLD}n_p^2}\right)(V_A - V_B) \quad [7\text{-}4g]$$

$$I_{pb} = \left(\frac{R_{LD}n_t^2+3R_{tLD}n_s^2}{3R_{LD}R_{tLD}n_p^2}\right)(V_B - V_C) \quad [7\text{-}5g]$$

$$I_{pc} = \left(\frac{R_{LD}n_t^2+3R_{tLD}n_s^2}{3R_{LD}R_{tLD}n_p^2}\right)(V_C - V_A)$$

[7-6g]

$$V_{sa} = \left(\frac{n_s}{n_p}\right)(V_A - V_B) \quad [7\text{-}7g]$$

$$V_{sb} = \left(\frac{n_s}{n_p}\right)(V_B - V_C) \quad [7\text{-}8g]$$

$$V_{sc} = \left(\frac{n_s}{n_p}\right)(V_C - V_A) \quad [7\text{-}9g]$$

$$I_{sa} = -\left(\frac{1}{R_{LD}}\right)\left(\frac{n_s}{n_p}\right)(V_A - V_B) \quad [7\text{-}10g]$$

$$I_{sb} = -\left(\frac{1}{R_{LD}}\right)\left(\frac{n_s}{n_p}\right)(V_B - V_C) \quad [7\text{-}11g]$$

$$I_{sc} = -\left(\frac{1}{R_{LD}}\right)\left(\frac{n_s}{n_p}\right)(V_C - V_A) \quad [7\text{-}12g]$$

$$V_{tsa} = \left(\frac{n_s}{n_p}\right)(V_A - V_B) \quad [7\text{-}26g]$$

$$V_{tsb} = \left(\frac{n_s}{n_p}\right)(V_B - V_C) \quad [7\text{-}27g]$$

$$V_{tsc} = \left(\frac{n_s}{n_p}\right)(V_C - V_A) \quad [7\text{-}28g]$$

$$V_{ta} = \left(\frac{n_t}{n_p}\right)(V_A - V_B) \quad [7\text{-}31g]$$

$$V_{tb} = \left(\frac{n_t}{n_p}\right)(V_B - V_C) \quad [7\text{-}32g]$$

$$V_{tc} = \left(\frac{n_t}{n_p}\right)(V_C - V_A) \quad [7\text{-}33g]$$

$$I_{ta} = -\left(\frac{1}{3R_{tLD}}\right)\left(\frac{n_t}{n_p}\right)(V_A - V_B) \quad [7\text{-}34g]$$

$$I_{tb} = -\left(\frac{1}{3R_{tLD}}\right)\left(\frac{n_t}{n_p}\right)(V_B - V_C) \quad [7\text{-}35g]$$

$$I_{tc} = -\left(\frac{1}{3R_{tLD}}\right)\left(\frac{n_t}{n_p}\right)(V_C - V_A) \quad [7\text{-}36g]$$

$$V_{tta} = \left(\frac{n_t}{n_p}\right)(V_A - V_B) \quad [7\text{-}40g]$$

$$V_{ttb} = \left(\frac{n_t}{n_p}\right)(V_B - V_C) \quad [7\text{-}41g]$$

$$V_{ttc} = \left(\frac{n_t}{n_p}\right)(V_C - V_A) \quad [7\text{-}42g]$$

$$I_{tlda} = \left(\frac{1}{R_{tLD}}\right)\left(\frac{n_t}{n_p}\right)V_A \quad [7\text{-}43g]$$

$$I_{tldb} = \left(\frac{1}{R_{tLD}}\right)\left(\frac{n_t}{n_p}\right)V_B \quad [7\text{-}44g]$$

$$I_{tldc} = \left(\frac{1}{R_{tLD}}\right)\left(\frac{n_t}{n_p}\right)V_C \quad [7\text{-}45g]$$

Figure 23 depicts the same circuit as Figure 22 with the addition of a ground fault on phase A of the secondary winding. Equations 7-126 and 7-130 are restored back to their original form:

$$I_{sa} + \frac{V_{tsa}}{R_{LD}} + I_{sfault} = 0 \quad [7\text{-}126]$$

$$V_{sng} + V_{tsa} = 0 \quad [7\text{-}130]$$

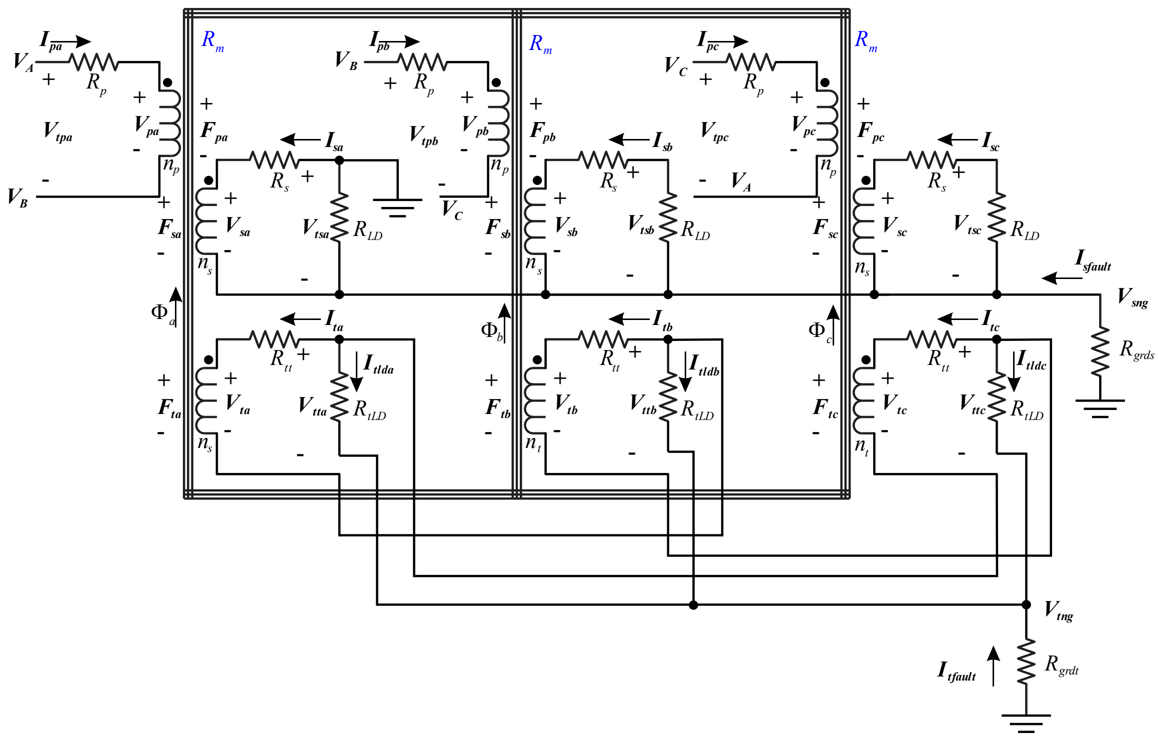

Figure 23: Three-phase transformer with two secondary windings – with resistive loads and ground fault

If we assume once again, that R_p, R_s, and R_{tt} are negligible, then the results are:

$$V_{pa} = (V_A - V_B) \quad [7\text{-}1h]$$

$$V_{pb} = (V_B - V_C) \quad [7\text{-}2h]$$

$$V_{pc} = (V_C - V_A) \quad [7\text{-}3h]$$

$$I_{pa} = \frac{(R_{LD}R_{grds}n_t{}^2 + 3R_{grds}R_{tLD}n_s{}^2 + 2R_{LD}R_{tLD}n_s{}^2)(V_A - V_B)}{3R_{LD}R_{grds}R_{tLD}n_p{}^2} \quad [7\text{-}4h]$$

$$I_{pb} = \frac{(R_{LD}R_{grds}n_t{}^2 + 3R_{grds}R_{tLD}n_s{}^2)(V_B - V_C) - R_{LD}R_{tLD}n_s{}^2(V_A - V_B)}{3R_{LD}R_{grds}R_{tLD}n_p{}^2} \quad [7\text{-}5h]$$

$$I_{pc} = \frac{(R_{LD}R_{grds}n_t{}^2 + 3R_{grds}R_{tLD}n_s{}^2)(V_C - V_A) - R_{LD}R_{tLD}n_s{}^2(V_A - V_B)}{3R_{LD}R_{grds}R_{tLD}n_p{}^2} \quad [7\text{-}6h]$$

$$V_{sa} = \left(\frac{n_s}{n_p}\right)(V_A - V_B) \quad [7\text{-}7h]$$

$$V_{sb} = \left(\frac{n_s}{n_p}\right)(V_B - V_C) \quad [7\text{-}8h]$$

$$V_{sc} = \left(\frac{n_s}{n_p}\right)(V_C - V_A) \quad [7\text{-}9h]$$

$$I_{sa} = -\left(\frac{n_s}{n_p}\right)\left(\frac{R_{grds} + R_{LD}}{R_{LD}R_{grds}}\right)(V_A - V_B) \quad [7\text{-}10h]$$

$$I_{sb} = -\left(\frac{n_s}{n_p}\right)\frac{(V_B - V_C)}{R_{LD}} \qquad [7\text{-}11h]$$

$$I_{sc} = -\left(\frac{n_s}{n_p}\right)\frac{(V_C - V_A)}{R_{LD}} \qquad [7\text{-}12h]$$

$$V_{ta} = \left(\frac{n_t}{n_p}\right)(V_A - V_B) \qquad [7\text{-}31h]$$

$$V_{tb} = \left(\frac{n_t}{n_p}\right)(V_B - V_C) \qquad [7\text{-}32h]$$

$$V_{tc} = \left(\frac{n_t}{n_p}\right)(V_C - V_A) \qquad [7\text{-}33h]$$

$$I_{ta} = -\left(\frac{n_t}{n_p}\right)\frac{(V_A - V_B)}{3R_{tLD}} \qquad [7\text{-}34h]$$

$$I_{tb} = -\left(\frac{n_t}{n_p}\right)\frac{(V_B - V_C)}{3R_{tLD}} \qquad [7\text{-}35h]$$

$$I_{tc} = -\left(\frac{n_t}{n_p}\right)\frac{(V_C - V_A)}{3R_{tLD}} \qquad [7\text{-}36h]$$

$$V_{sng} = -\left(\frac{n_s}{n_p}\right)(V_A - V_B) \qquad [7\text{-}29h]$$

$$I_{sfault} = \left(\frac{n_s}{n_p}\right)\frac{(V_A - V_B)}{R_{grds}} \qquad [7\text{-}30h]$$

Conclusions include:

a) A ground fault on the secondary winding does not impact the tertiary winding
b) The primary currents are impacted by the ground fault – their magnitudes are not equal and their angles are not separated by 120˚

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